

Flavorful Instantons and the Strong CP Problem

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UW Madison HEP Seminar, Oct. 4 2016

Based on work in progress w/Prateek Agrawal

Strong CP Problem

$$\mathcal{L} = \theta G\tilde{G} + y_u H Q u^c + y_d H Q d^c$$

Two SM CP violating Phases:

Strong CP
(non-perturbative)

$$\bar{\theta} = \theta - \text{Argdet}y_u - \text{Argdet}y_d$$

CKM

$$V_{CKM} \supset e^{i\delta_{CKM}}$$

Puzzle: CKM Phase $\mathcal{O}(1)$ but $\bar{\theta} \lesssim 10^{-10}$
(neutron EDM)

Sequestering?

SM is special $\Delta\bar{\theta} \ll \delta_{CKM}$

Massless Up Quark Solution

Two-Flavor (single generation) QCD:

$$\mathcal{L} = \theta \tilde{G}_{\mu\nu} G^{\mu\nu} + m_u uu^c + m_d dd^c + \text{h.c.}$$

$$\left[\begin{array}{l} \text{chiral} \\ \text{rotation:} \\ \text{Axial anomaly:} \end{array} \begin{array}{l} uu^c \rightarrow e^{i\alpha} u' u'^c \\ dd^c \rightarrow e^{i\beta} d' d'^c \\ \theta \rightarrow \theta + \alpha + \beta \end{array} \right] \left. \begin{array}{l} \text{Invariant CP Phase} \\ \bar{\theta} = \theta - \text{Arg}m_u - \text{Arg}m_d \end{array} \right]$$

$m_u = 0 \longrightarrow U(1)_{PQ} \longrightarrow$ No Strong CP violation

Anomalous \longrightarrow Non-perturbative QCD
generates up mass

$$m_u \neq 0?$$

Massless Up Quark Solution

Dilute Instanton Gas Approximation



instanton "density" $D(\rho)$

$$\Delta m_u = \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} \times C_0 \times \left(\frac{8\pi^2}{g(\rho)^2} \right)^6 e^{-\frac{8\pi^2}{g(\rho)^2}} \times \underbrace{m_d^* e^{i\theta}}_{\text{invariant phase}}$$

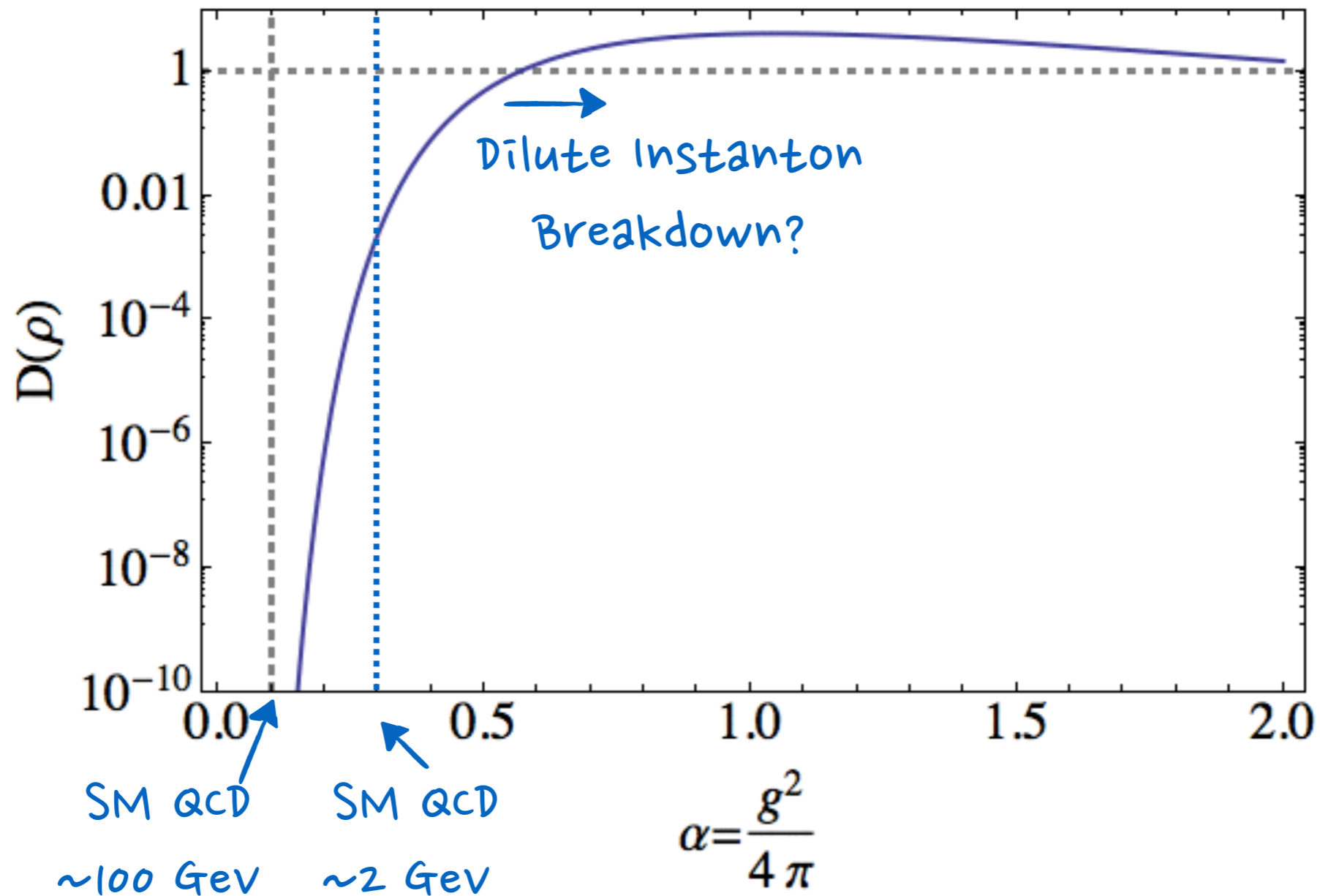
Instanton sizes

invariant phase

$$\arg(\Delta m_u) + \arg(m_d) - \theta = 0$$

Massless Up Quark Solution

instanton "density" $D(\rho) = C_0 \times \left(\frac{8\pi^2}{g(\rho)^2} \right)^6 e^{-\frac{8\pi^2}{g(\rho)^2}}$



Massless Up Quark Solution

Protection from UV
Effects?

$$\Delta m_u = \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} \times D(\rho) \times m_d(\rho)^* e^{-i\theta}$$

running CP phase of mass?

Λ_{UV}
unsequestered
CP violation

$$m'_d = e^{i\delta} m_d$$

$$\Delta m'_u \sim D(\Lambda_{UV}^{-1}) m_d^* e^{-i\delta}$$

$$m_u = \Delta m_u + \Delta m'_u$$

→ phase mismatch

$$\Delta \bar{\theta} \sim D(\Lambda_{UV}^{-1}) \frac{|m_d|}{|m_u|} \times \delta$$

Λ_{QCD}

$$m_d$$

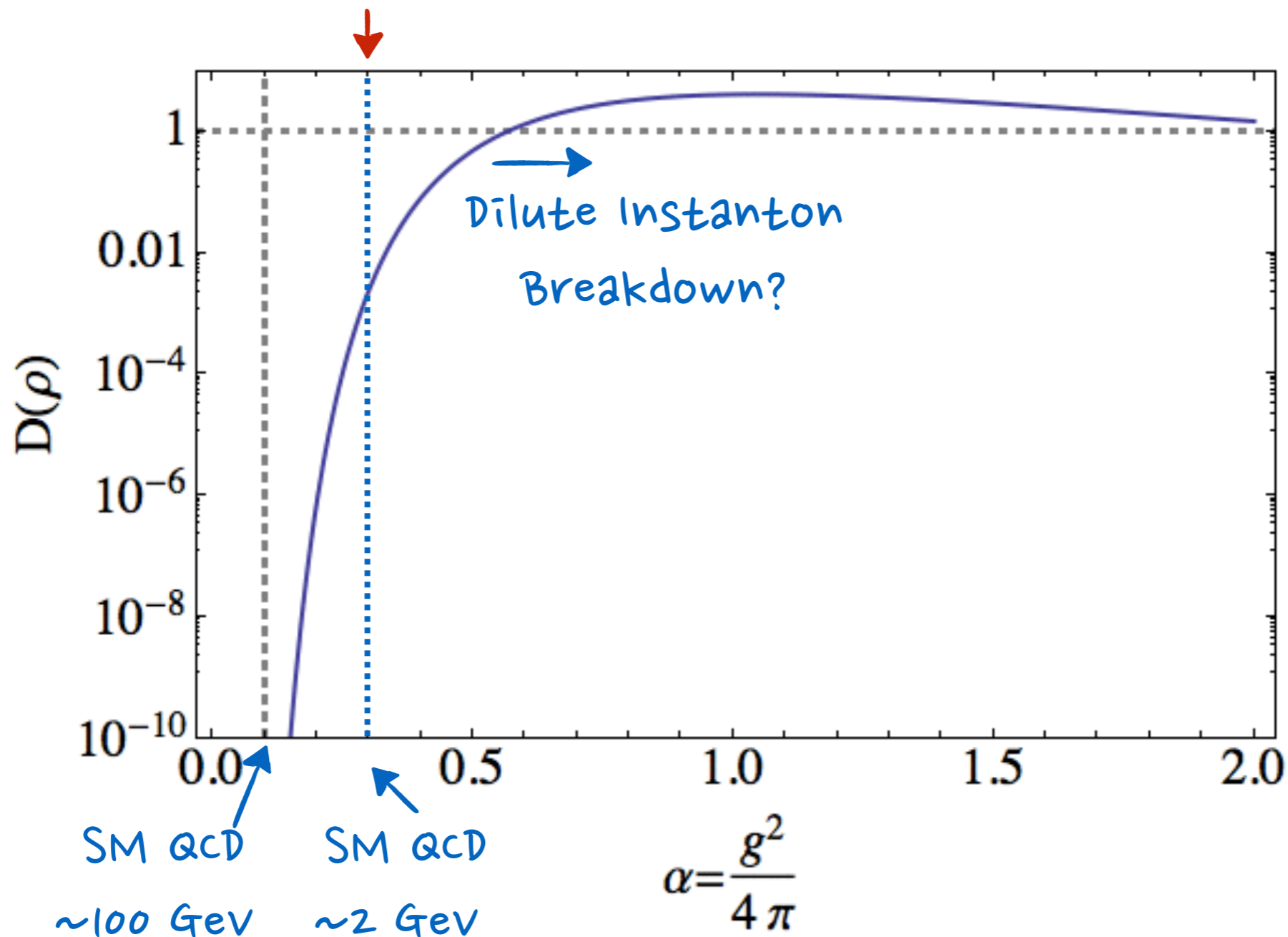
$$\Delta m_u \sim D(\Lambda_{QCD}^{-1}) m_d^*$$

Suppression of extra PQ Breaking?

$$m_u = \Delta m_u + \Delta m'_u + m_u^0 \quad \Delta \bar{\theta} \sim |m_u^0| / |m_u|$$

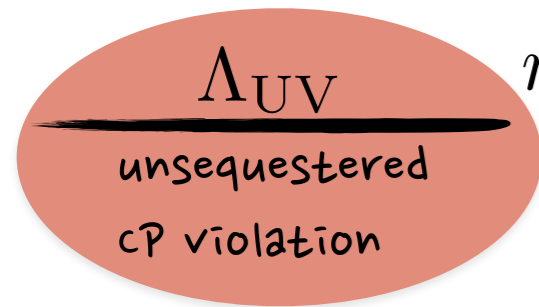
Massless Up Quark Solution

Lattice
cutoff: $m_u^0 \approx 0.5m_d^0 \rightarrow$ massless up quark solution fails?

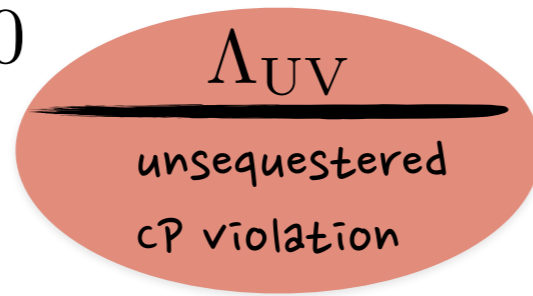


(chiral Lagrangian: higher order terms correspond to instantons, too small)

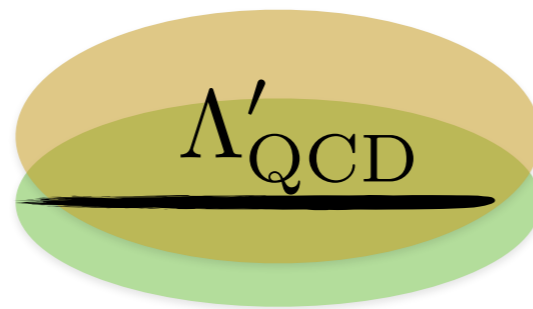
UV Instantons and PQ?



$$m_u \approx 0$$



$$m_u \approx 0$$

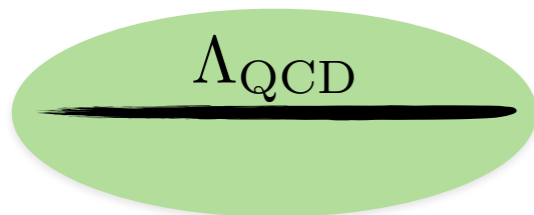


CP-violation remains sequestered

Non-perturbative effects important again



$$\Delta m_u \sim m_d^* e^{i\delta}$$



Normal



UV Instanton solutions

One 'wrong' way

~~Λ_{UV}~~

$SU(3 + N)$

~~Λ'_{QCD}~~

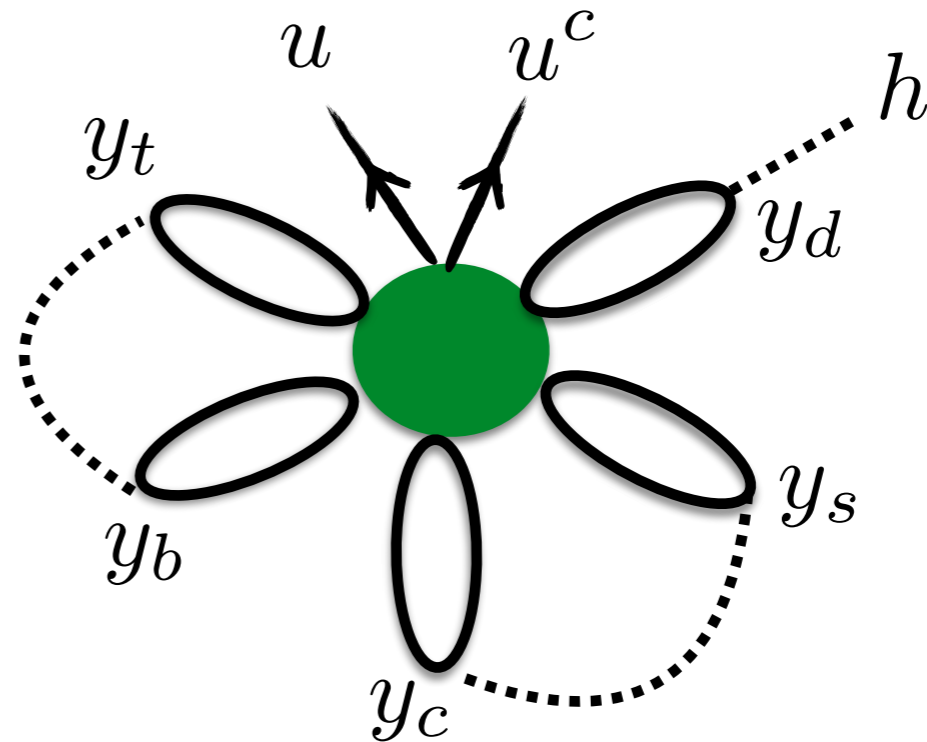
Higgsing

(+ any extra chiral symmetries strongly broken)

SM + Extra Matter

~~_____~~

~~Λ_{QCD}~~

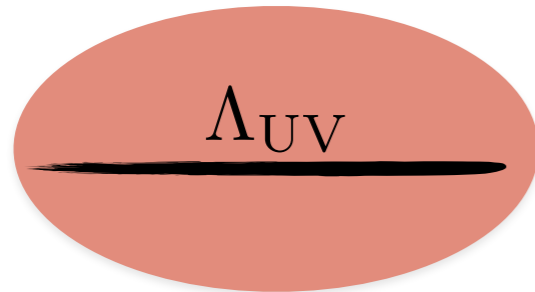


$$\frac{\Delta y_u}{y_d} \sim D(\Lambda'_{QCD}) \frac{y_t y_b y_s y_c}{(16\pi^2)^2}$$

too small

(extra breaking of SM chiral symmetries typically introduces phases)

Flavorful Instantons



$SU(3) \times SU(3) \times SU(3) (\times \dots)$

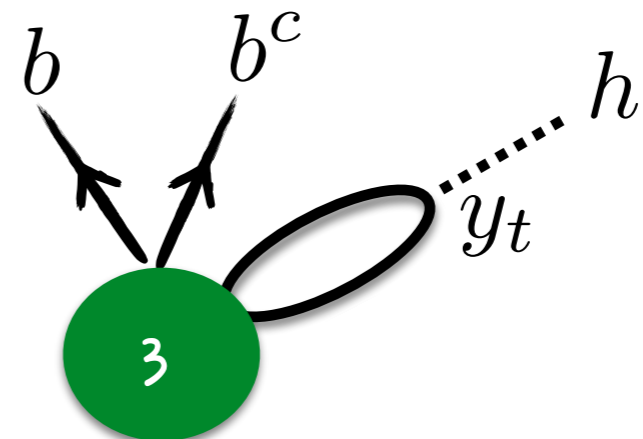
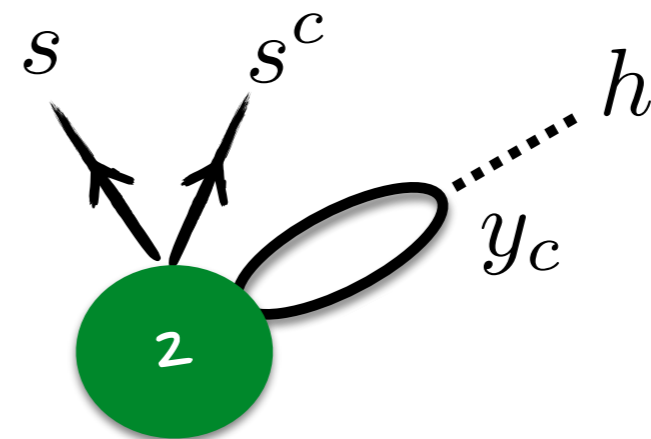
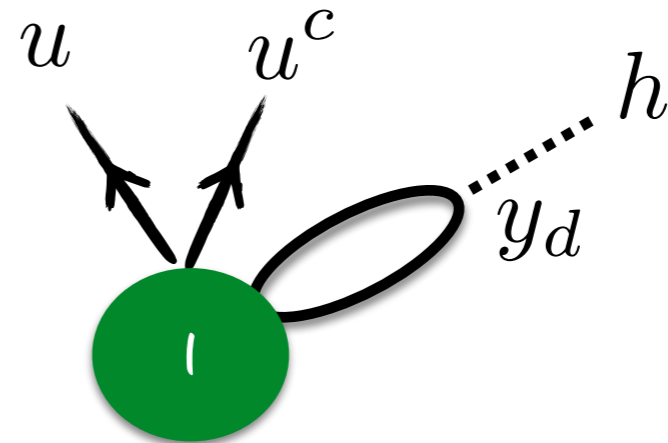
coincidence of scales?



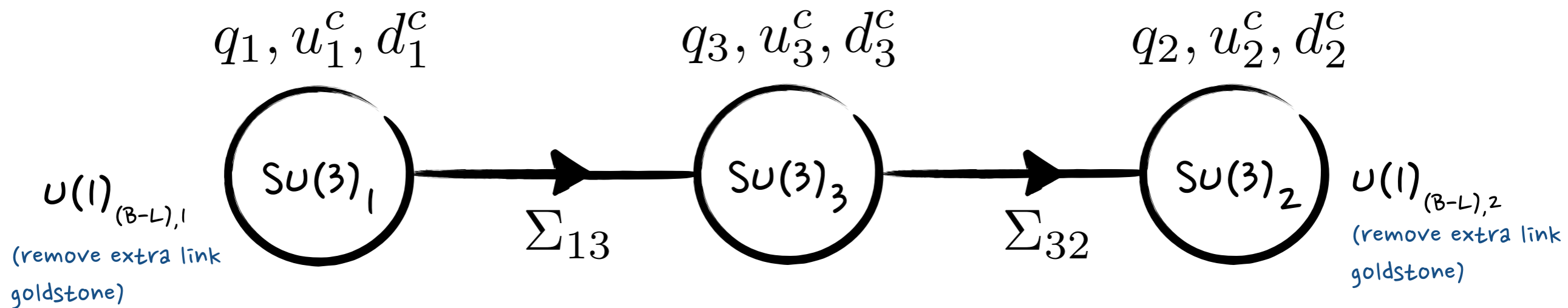
Higgsing

(+ any extra chiral symmetries strongly broken)

SM



3x(2-flavor) models



$2Q \times 3$ [

$$y_u = 0$$

$$y_b = 0$$

$$y_s = 0$$

$$V(\Sigma_{13}) = (|\Sigma_{13}|^2 - v_{13}^2)^2$$

$$V(\Sigma_{32}) = (|\Sigma_{32}|^2 - v_{32}^2)^2$$

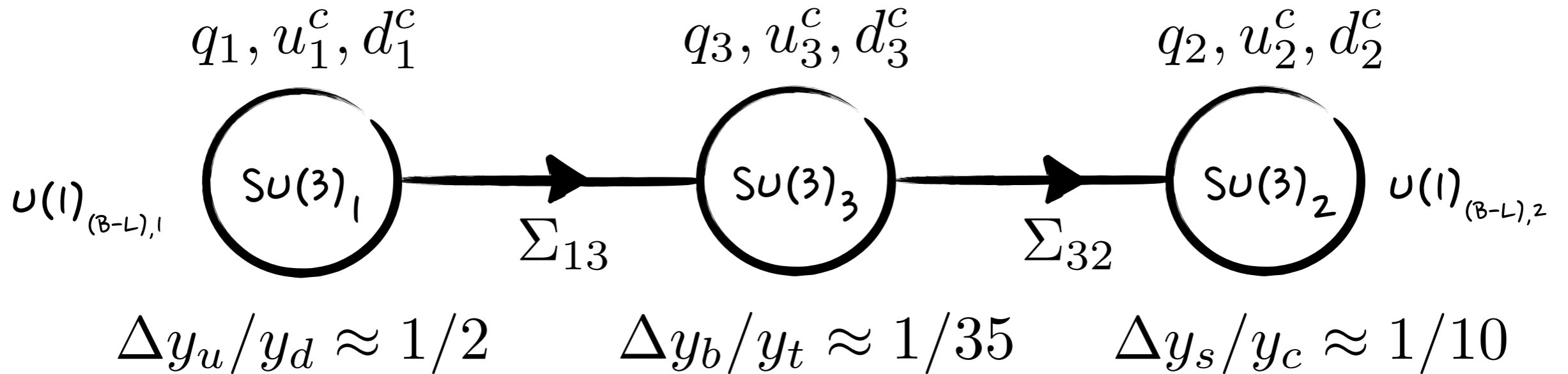
Higgsing [

$$SU(3)_1 \times SU(3)_2 \times SU(3)_3 \rightarrow SU(3)_{\text{QCD}}$$

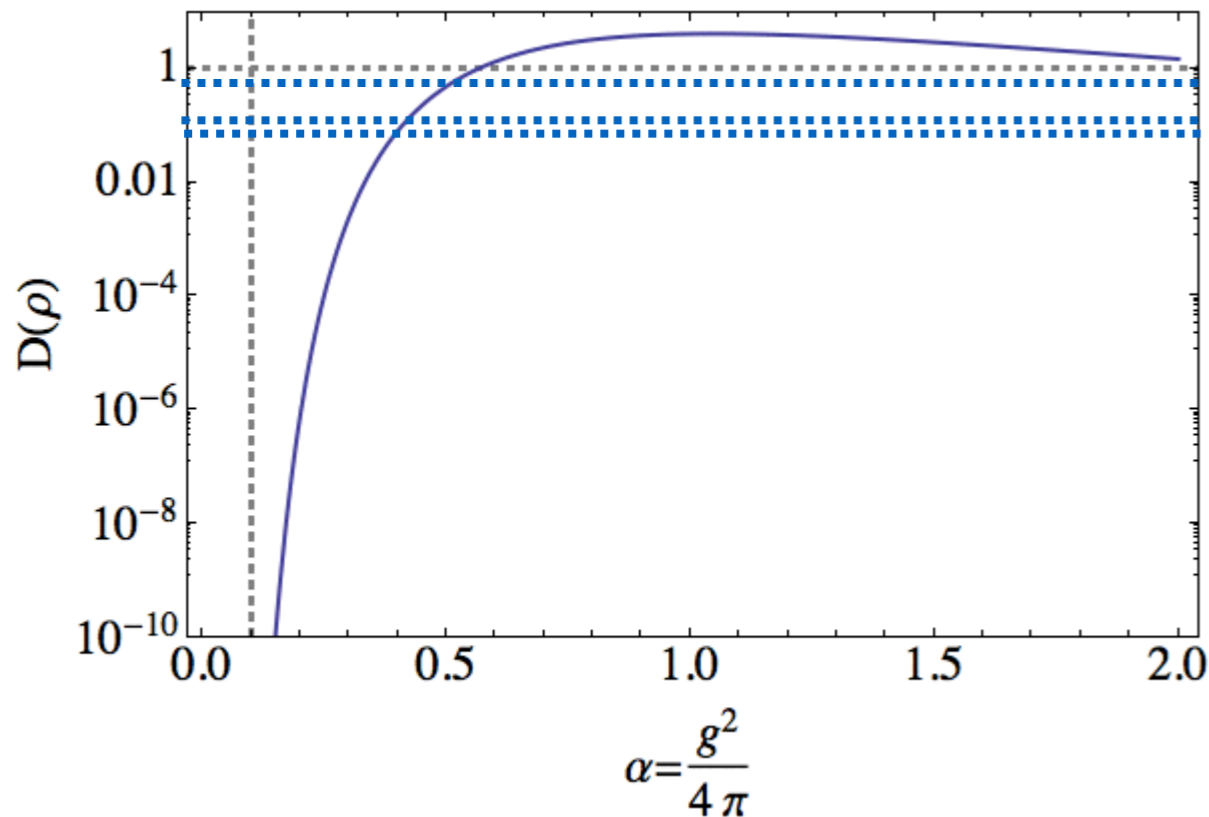
Strongly coupled \longrightarrow weakly coupled

$$\Lambda'_{\text{QCD}} \sim gv_{13}, gv_{32}$$

3x(2-flavor) models

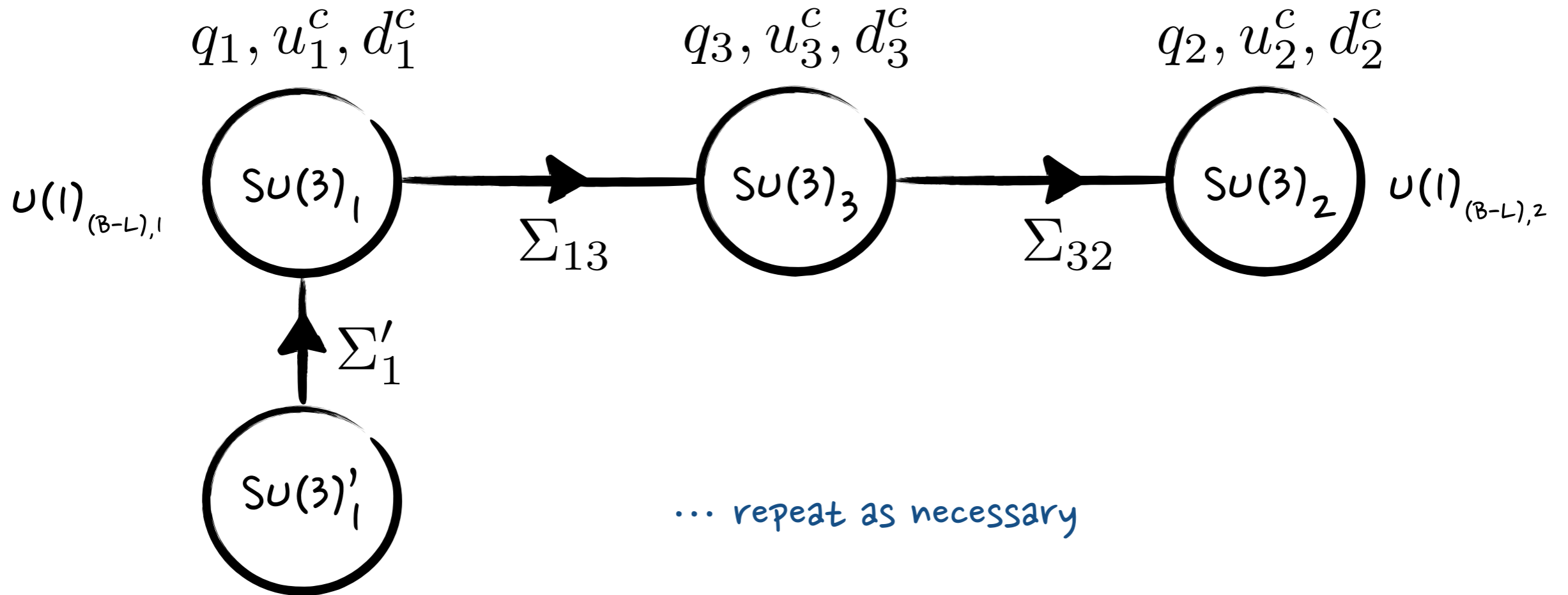


instanton density



$$\frac{1}{\alpha_s} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \longrightarrow \alpha_s \gtrsim 0.2$$

weak coupling



e.g. vector-like matter w/PQ

$$\psi'_1, \bar{\psi}'_1 \quad m_\psi = 0$$

$$\Delta m_\psi = D(gv'_1) \times (gv_1) \sim 10^{-6} \Lambda_{\text{QCD}'}$$

CKM

Above instanton scale:

(background fields)

Higher dim $\sim \frac{\langle \Sigma_{13} \rangle \langle \Sigma_{32} \rangle}{\Lambda^2}$

only 1 phase
(depends on 4 elements)

Anomalous $U(1)$'s
remove theta

$$(Q_i y_u^{ij} u_j^c) H$$

$$(Q_i y_d^{ij} d_j^c) H^\dagger$$

		$U(1)_{u_1}$	-1	-1		$U(1)_{d_2}$	-1
$U(1)_{q12}$	+1	0	$y_c s_{\theta_{12}}$	0	y_d	0	$V_{31} y_b e^{i\delta_0}$
	+1	0	$y_c c_{\theta_{12}}$	0	$\tilde{y}_d \sim 0$	0	$V_{32} y_b$
$U(1)_{q3}$	+1	0	0	y_t	0	0	0

Spurion/radiative:

$$\tilde{y}_d \sim y_d \times \frac{1}{16\pi^2} (\sim y_b^2 V_{31} V_{32} + \sim y_c^2 s_{\theta_{12}} c_{\theta_{12}})$$

Why this structure?

Need at least 3 orthogonal anomalous $U(1)$ to remove phases

CKM

Below instanton scale:

$$(Q_i y_u^{ij} u_j^c) H$$

$$\begin{pmatrix} \frac{y_u}{c\theta_{12}} & y_c s\theta_{12} & 0 \\ 0 & y_c c\theta_{12} & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$(Q_i y_d^{ij} d_j^c) H^\dagger$$

$$\begin{pmatrix} y_d & 0 & V_{31} y_b e^{i\delta_0} \\ \tilde{y}_d \sim 0 & y_s & V_{32} y_b \\ 0 & 0 & y_b \end{pmatrix}$$

$\sim 1/100$ (arrow to $V_{31} y_b e^{i\delta_0}$)
 $\sim \pi/8$ (arrow to $e^{i\delta_0}$)
 $\sim 1/25$ (arrow to y_b)

Why this structure?

Need at least 3 orthogonal anomalous $U(1)$ to remove phases

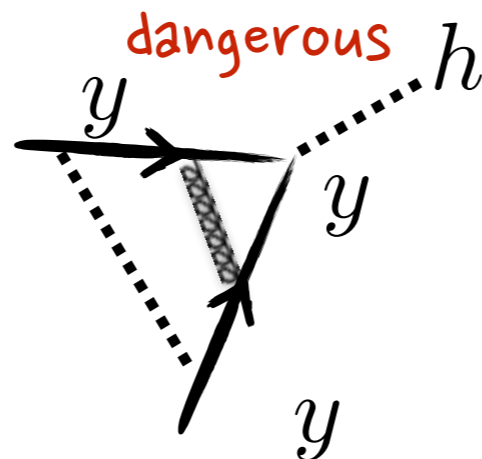
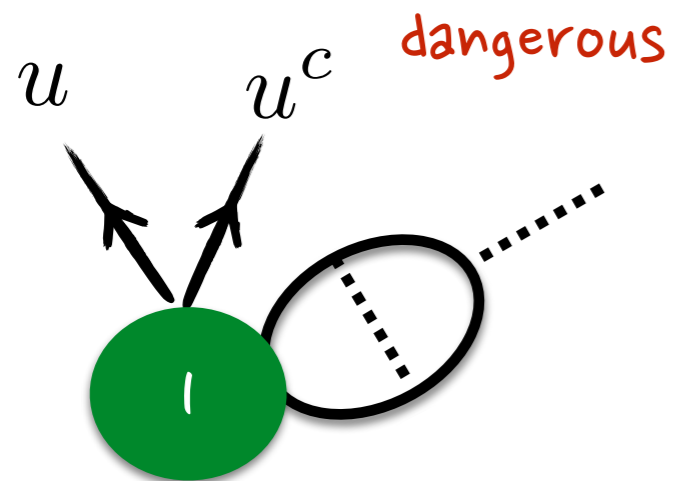
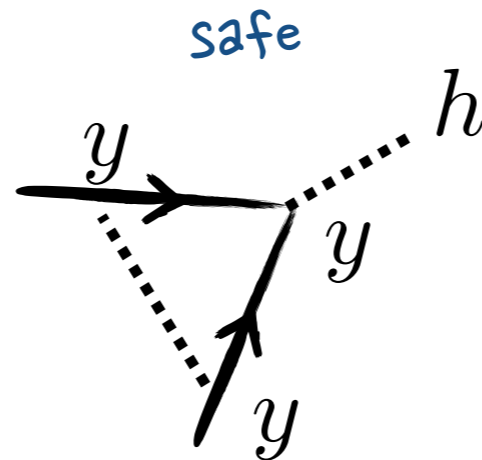
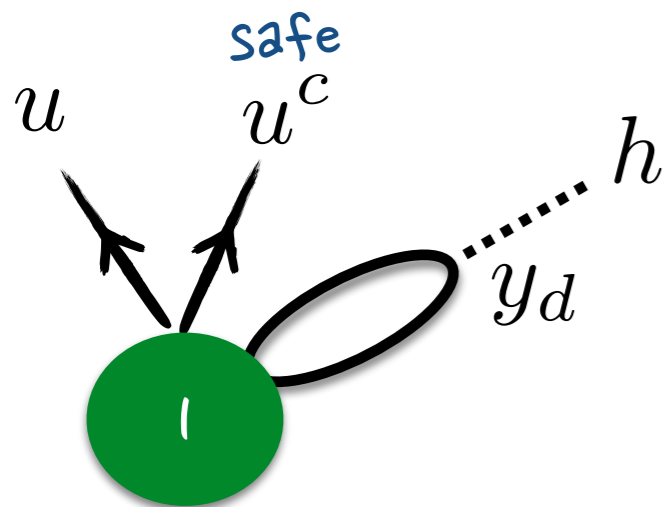
Radiative Sequestering of CPV?

Looks a lot like the SM...

BUT $SU(3) \times SU(3) \times SU(3)$ breaks many of the flavor symmetries.

running CP phase of mass?

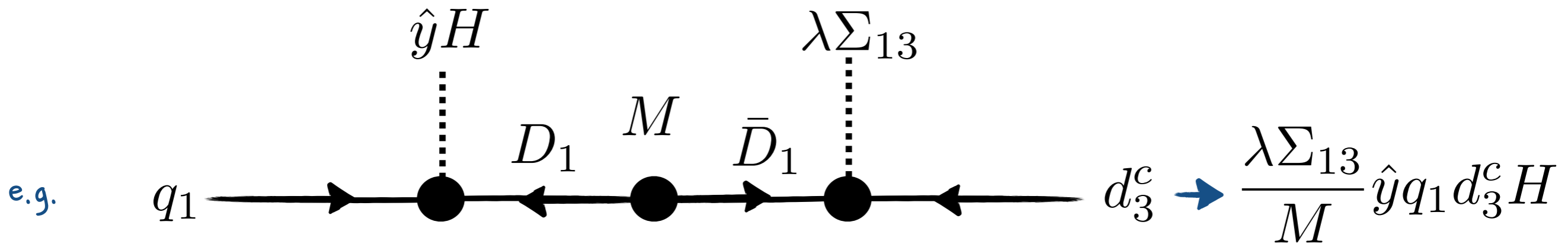
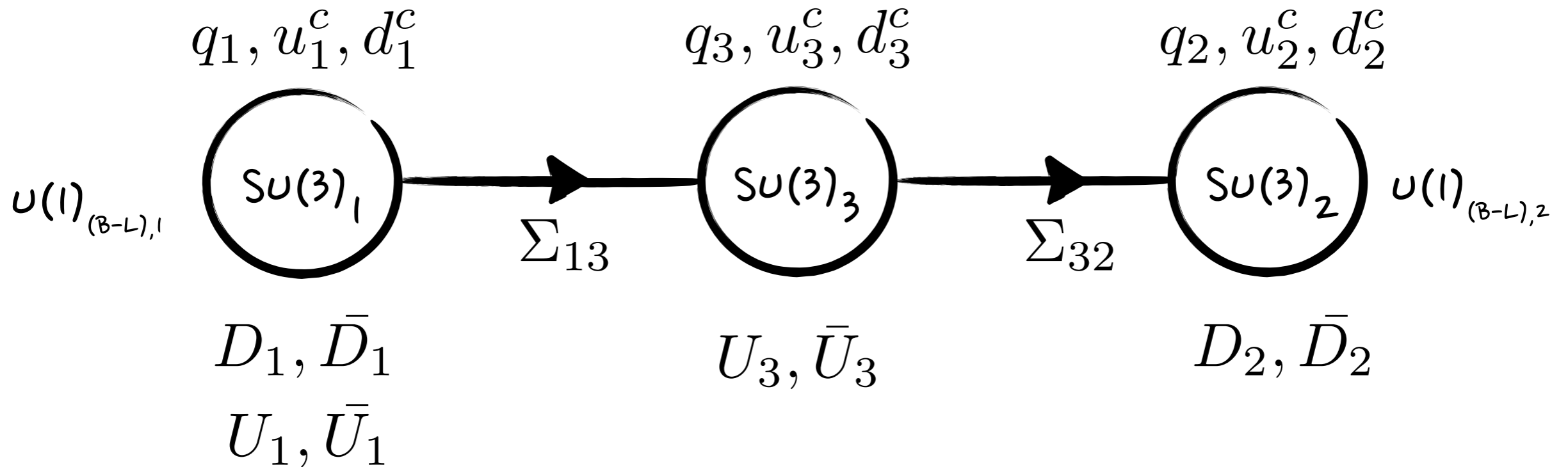
$$\Delta m_u = \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} \times D(\rho) \times m_d(\rho)^* e^{-i\theta}$$



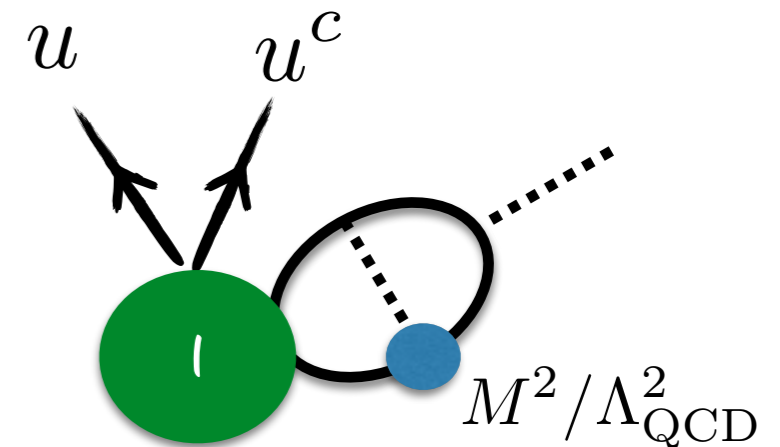
$$\Delta \bar{\theta} \approx \frac{6}{64\pi^2} \sin \delta_0 \tan \theta_{12} V_{31} V_{32} y_b^2$$

$$\approx 2 \times 10^{-10}$$

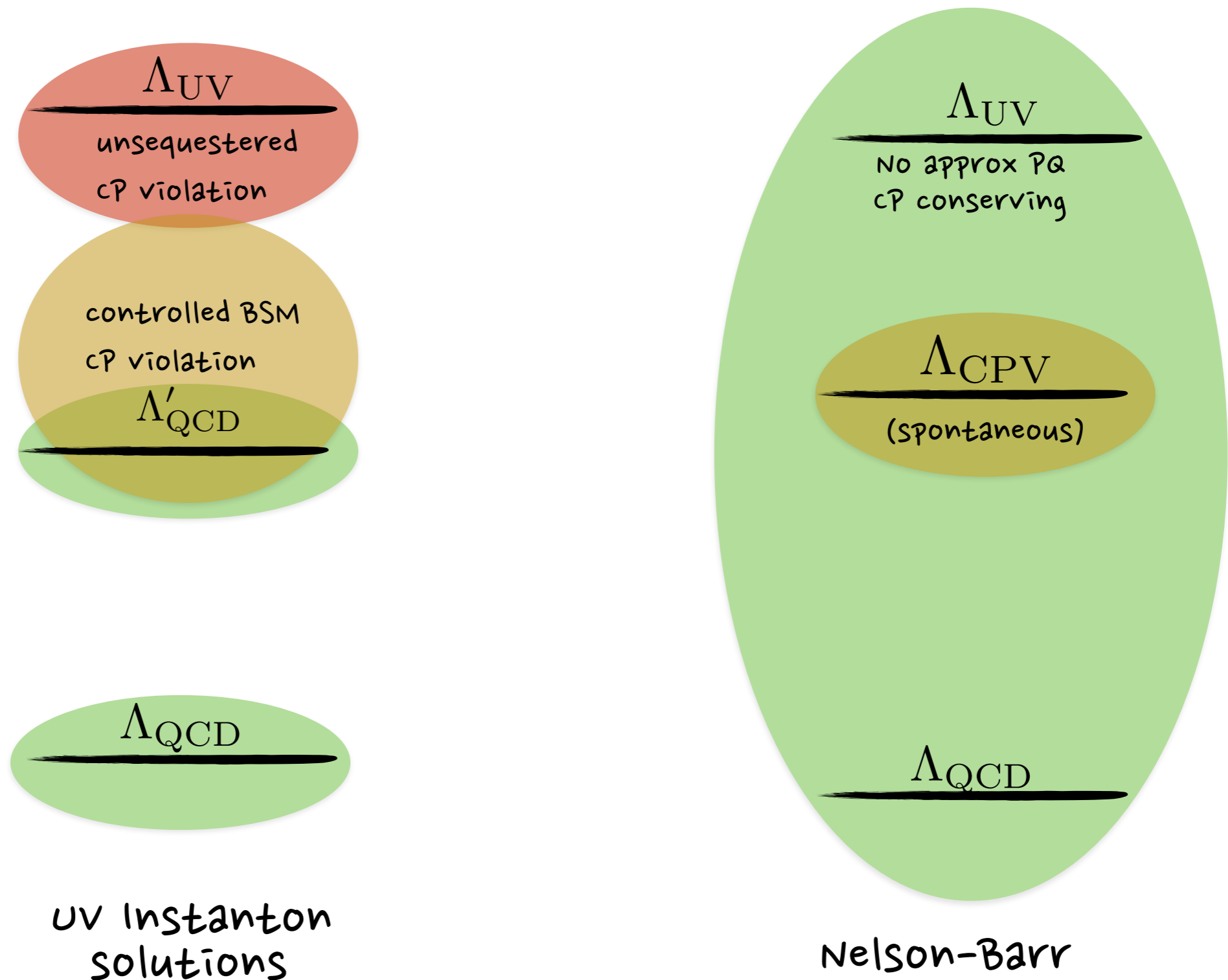
Flavor Completion



soft flavor? $M \lesssim \Lambda_{\text{QCD}'}$ \longrightarrow soft CP violation



Radiative Phases



Alternative hierarchy

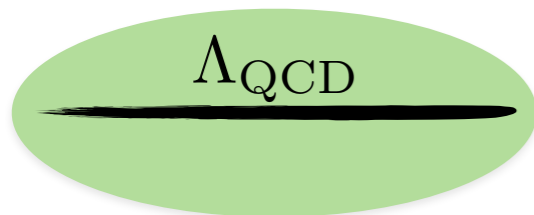
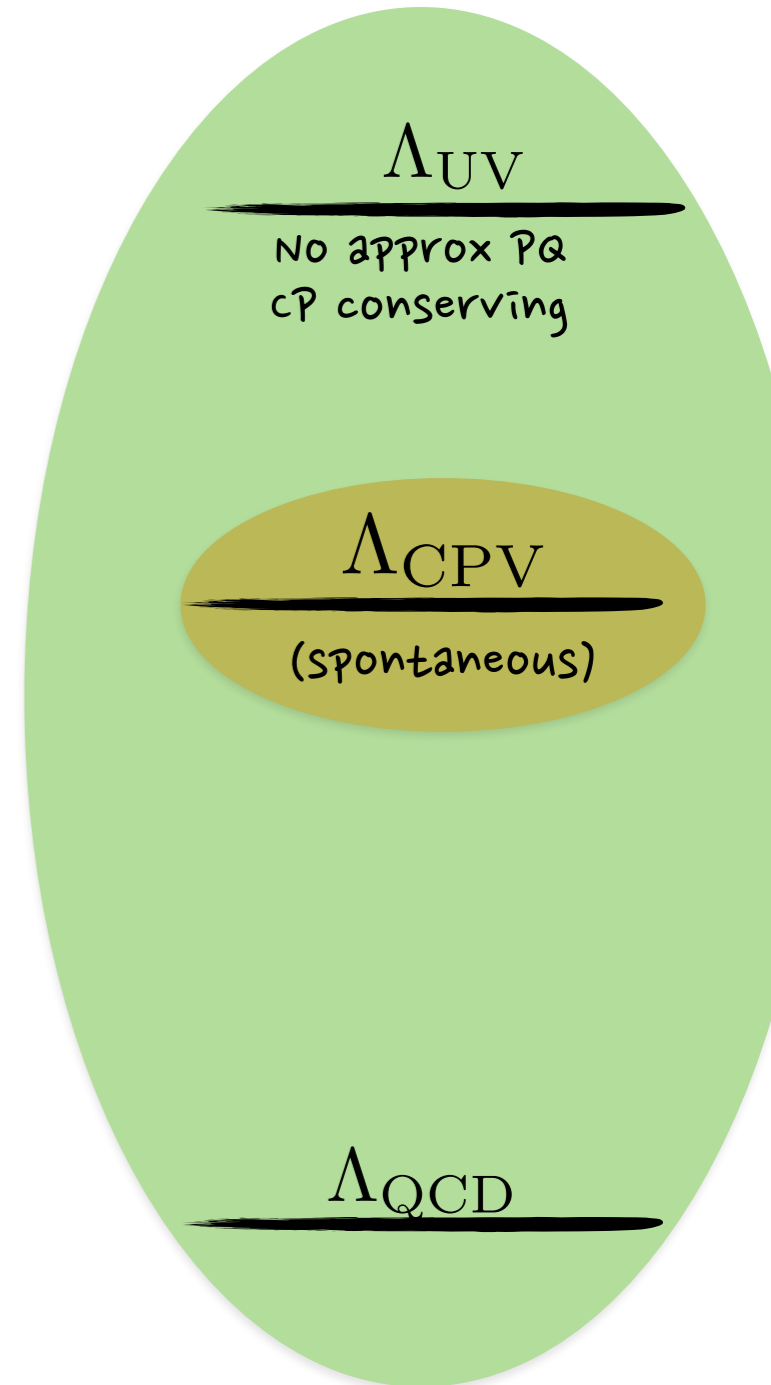
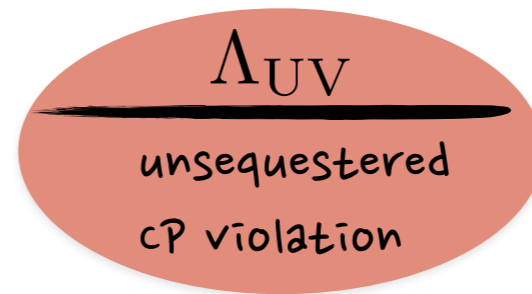
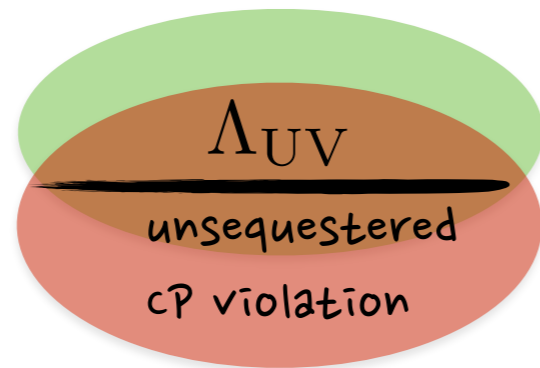
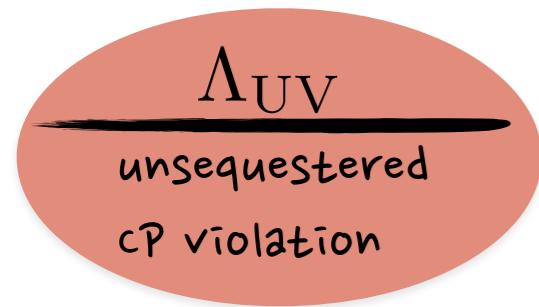
$$\begin{pmatrix} 0 & y_c s\theta_{12} & 0 \\ 0 & y_c c\theta_{12} & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \begin{pmatrix} y_d & 0 & V_{31} y_b e^{i\delta_0} \\ \tilde{y}_d \sim 0 & 0 & V_{32} y_b \\ 0 & 0 & 0 \end{pmatrix}$$

$$y_d \rightarrow y_s, s\theta_{12} \rightarrow c\theta_{12}, V_{31} \rightarrow V_{32}$$

$$\Delta y_u \sim y_s, \Delta y_d \sim y_c \quad \text{light quark masses from instantons!}$$

$$\Delta \bar{\theta} \sim 3 \times 10^{-9} \quad \rightarrow \text{soft flavor or tuning}$$

Conclusions



Normal

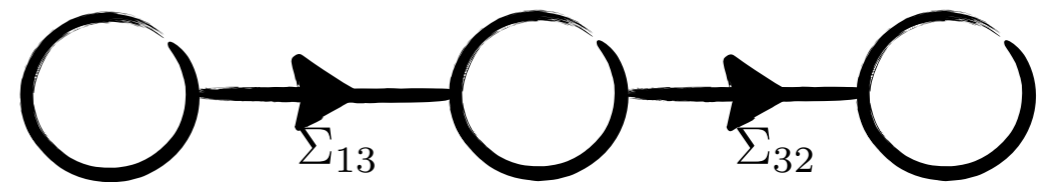
Dangerous

UV Instanton
solutions

Nelson-Barr

Conclusions

- Mechanism for high-scale aligned PQ breaking effects
 - massless quark OR axion
 - $U(1)_{PQ}$ same status as axion solutions
 - Decoupling — no light states
 - Different flavor from Z_2 or extra IR running
 - Not spontaneous CP violation



- Further directions...
 - connection to weak scale? (SUSY)
 - flavor observables
 - accidental scales?
 - axions, axions + fermions, etc.
 - 5D picture

- Flavor Model
 - Instantons explain some hierarchies
 - Preserves sequestering of SM
 - observable size for strong CP?

$$\begin{pmatrix} \frac{y_u}{c_{\theta_{12}}} & y_c s_{\theta_{12}} & 0 \\ 0 & y_c c_{\theta_{12}} & 0 \\ 0 & 0 & y_t \end{pmatrix} \begin{pmatrix} y_d & 0 & V_{31} y_b e^{i\delta_0} \\ \tilde{y}_d \sim 0 & y_s & V_{32} y_b \\ 0 & 0 & y_b \end{pmatrix}$$

Thanks!

back-ups
