

The Fate of Axion Stars



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PRL 117, 121801 (2016) PRD 94, 076004 (2016)

arXiv:1609.05182

Outline

✧ Axions

✧ (Dilute) Axion Star

✧ Dense Axion Star

PRL 117, 121801 (2016)

✧ Observables

arXiv:1609.05182

✧ Axion EFT

PRD 94, 076004 (2016)

✧ Summary

Axions

- Peccei-Quinn U(1) symmetry solves **strong CP problem**

Peccei & Quinn (1977)

- Introduces a Goldstone boson -- **Axion**

Weinberg (1978), Wilczek (1978)

- Strongly motivated candidate for **cold dark matter**.

Lect. Notes Phys. 741 (2008)

A recent review: Kim & Carosi (2010)

Relativistic Axions

Real pseudoscalar field

Energy scale below 1 GeV

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mathcal{V}(\phi)$$

Two models for potential

- **Instanton** $\mathcal{V}(\phi) = m_a^2 f_a^2 [1 - \cos(\phi/f_a)]$

m_a : axion mass

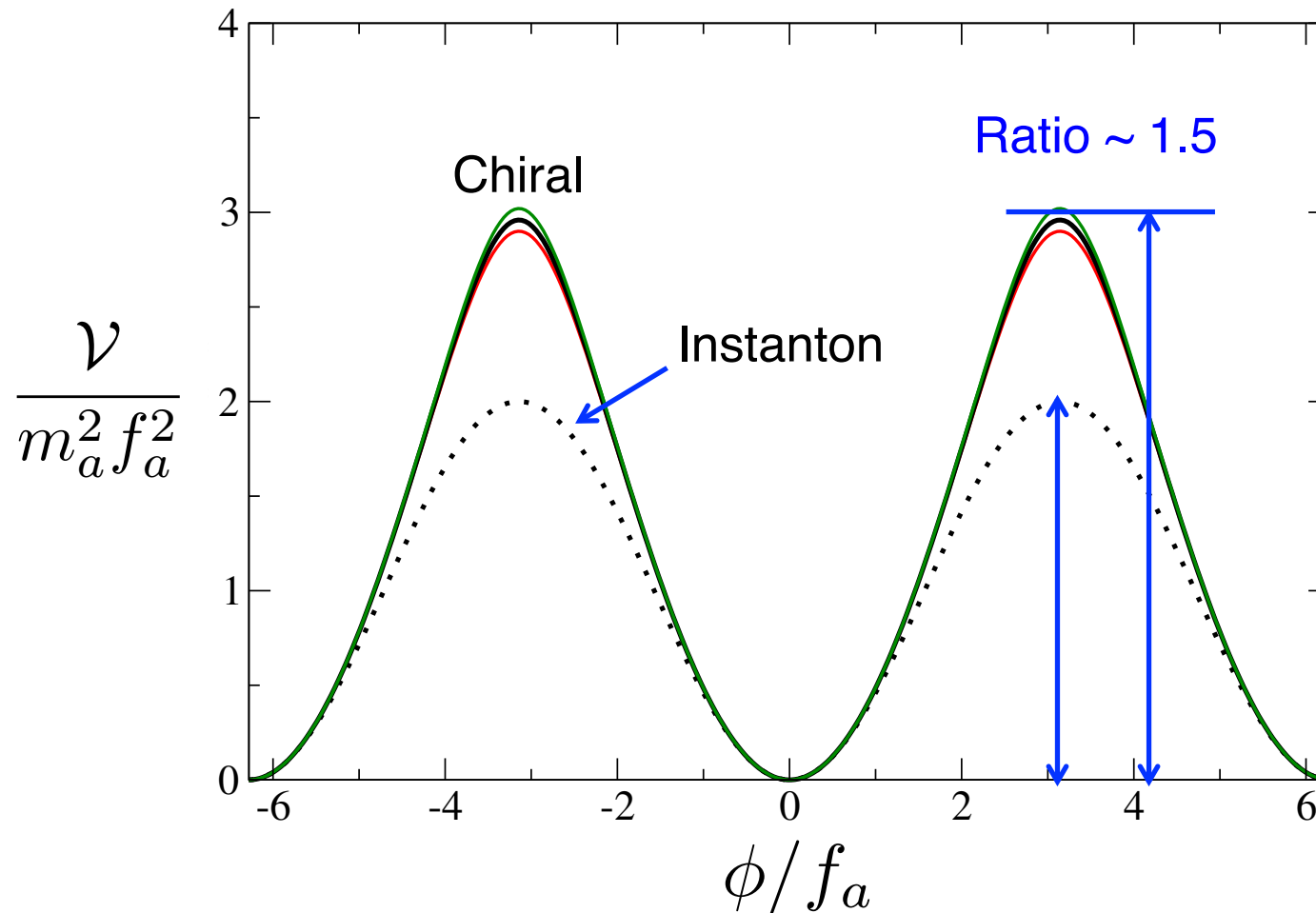
f_a : axion decay constant

- **Chiral** $\mathcal{V}(\phi) = m_\pi^2 f_\pi^2 \left(1 - \left[1 - \frac{4z}{(1+z)^2} \sin^2(\phi/2f_a) \right]^{1/2} \right)$

$$z = m_u/m_d \approx 0.48$$

Relativistic Axion Potential

Periodic potentials $\mathcal{V}(\phi) = \mathcal{V}(\phi + 2\pi f_a)$



Parameters & Current Constraints

- Two parameters in relativistic axion Lagrangian:

$$m_a \text{ and } f_a$$

- Not independent, related by QCD

$$m_a^2 f_a^2 = \frac{z}{(1+z)^2} m_\pi^2 f_\pi^2 \quad \longrightarrow \quad m_a f_a = (80 \text{ MeV})^2$$

$$z = m_u/m_d \approx 0.48$$

- Constraints from astrophysics & cosmology

$$10^8 \text{ GeV} < f_a < 10^{13} \text{ GeV} \quad \longrightarrow \quad 10^{-6} \text{ eV} < m_a < 10^{-2} \text{ eV}$$

Very weak self-interaction !

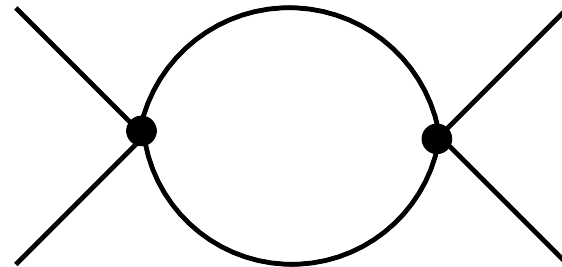
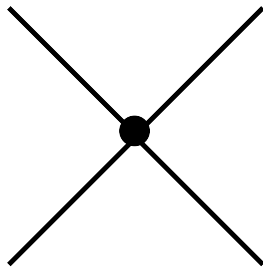
Tiny Mass !!

In this talk, I choose $m_a = 10^{-4} \text{ eV}$

Loop Contribution is Small

Each loop is suppressed by

$$(m_a/f_a)^2 \sim 10^{-48}$$



- Diagrams with loops can be safely ignored.

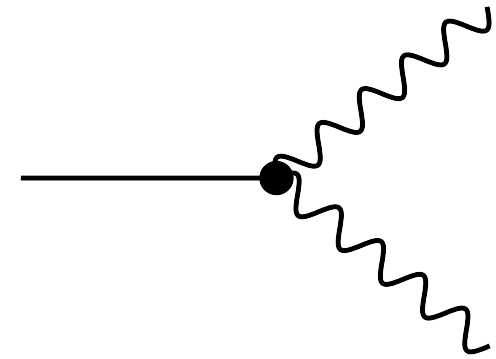
Axion-Photon Coupling

- Very weak coupling

$$\mathcal{L}_{\text{em}} = \frac{c_{\text{em}}\alpha}{16\pi f_a} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \phi.$$

$$c_{\text{em}} \sim 1 \quad \text{Model dependent}$$

Suppressed by $f_a \sim 10^{11} \text{ GeV}$



- Decay rate into two photons

$$\Gamma_a = \frac{c_{\text{em}}^2 \alpha^2 m_a^3}{256\pi^3 f_a^2}.$$

Axion lifetime $\sim 10^{36}$ years
Age of Universe $\sim 10^{10}$ years

Photon energy: $m_a/2 \sim 10 \text{ GHz}$ **Radio frequency**

Axion Cosmology

- Cold dark matter axions are produced **abundantly** at QCD phase transition scale $T \sim 1 \text{ GeV}$

Non-relativistic axion production mechanism

For more details, see Lect. Notes Phys. 741 (2008)

➤ Vacuum misalignment

Coherent

Preskill, Wise & Wilczek (1983)

Abbot & Sikivie (1983)

Dine & Fischler (1983)

➤ Cosmic string decay

Incoherent

Davis (1986)

Hararie & Sikivie (1987)

Occupation number $n_a \lambda_{dB}^3 |_{T=1\text{GeV}} \approx 10^{58}$

Sikivie & Yang PRL (2009)

Form Bose-Einstein condensate if can be effectively thermalized

Gravitational Thermalization

- Axion self-interaction may be too weak to thermalize axions
- **Gravitational interaction** can thermalize axions

Sikivie & Yang PRL (2009)

- Bring initially **incoherent** axions into **coherence**
- Keep the axion field as a **Bose-Einstein Condensate** as the Universe evolves

- Correlation length

Galactic scale? Sikivie & Yang PRL (2009)

Stellar scale due to **attractive** self-interaction?

Guth, Hertzberg & Prescod-Weinstein PRD (2015)

Is there a (meta)stable axion star solution?

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✧ (Dilute) Axion Star

✧ Dense Axion Star

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arXiv:1609.05182

✧ Axion EFT

PRD 94, 076004 (2016)

✧ Summary

Non-relativistic EFT (Part I)

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mathcal{V}(\phi)$$

Real Scalar

Chavanis PRD (2011)

Chavanis & Delfini PRD (2011)

Instanton potential / Chiral potential

Naïve non-relativistic reduction

Complex scalar

$$\phi(\mathbf{r}, t) = \frac{1}{\sqrt{2m_a}} \left[\psi(\mathbf{r}, t) e^{-im_a t} + \psi^*(\mathbf{r}, t) e^{+im_a t} \right]$$

Ignore all terms with rapid oscillating phase

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} i \left(\psi^* \dot{\psi} - \dot{\psi}^* \psi \right) - \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi - \mathcal{V}_{\text{eff}}$$

$$\mathcal{V}_{\text{eff}} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4} + \dots$$

Expand by $\frac{\psi^* \psi}{m_a f_a^2}$

Non-relativistic EFT (Part I)

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Attractive interaction!

Expand by $\frac{\psi^* \psi}{m_a f_a^2}$

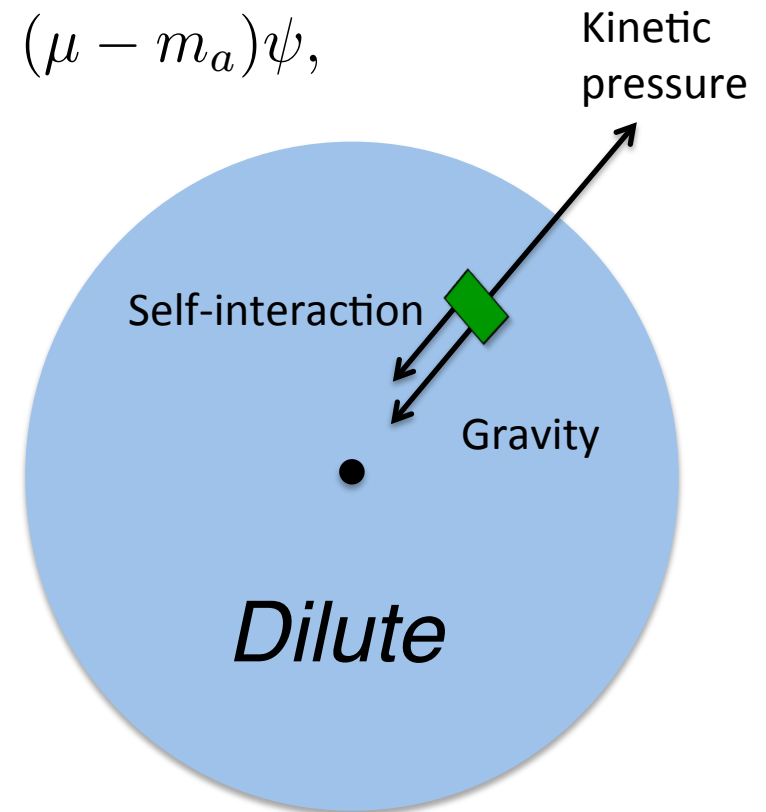
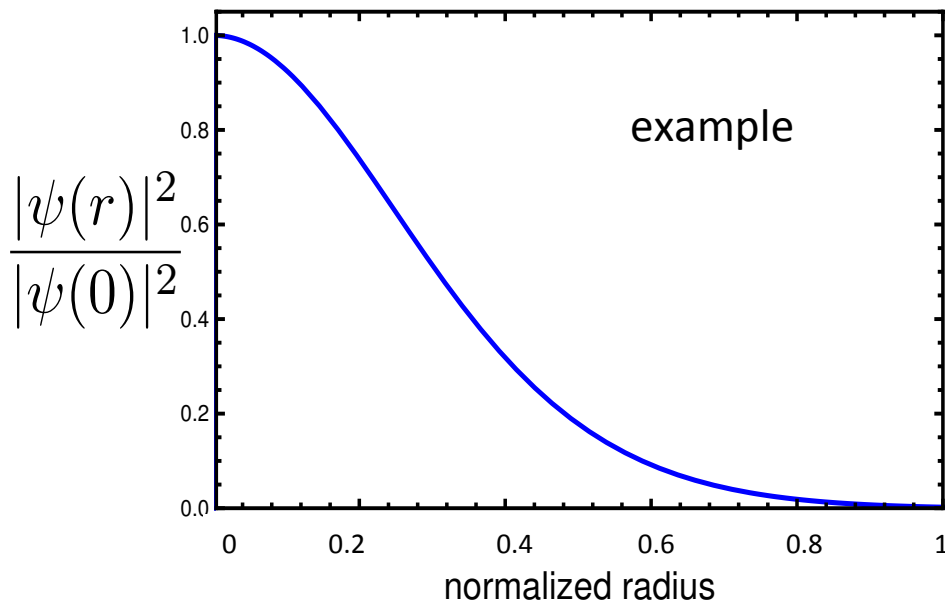
Dilute limit

Dilute Axion Stars

- Assume:
- Instanton potential, dilute axion limit
 - Newtonian gravity
 - Spherically symmetric

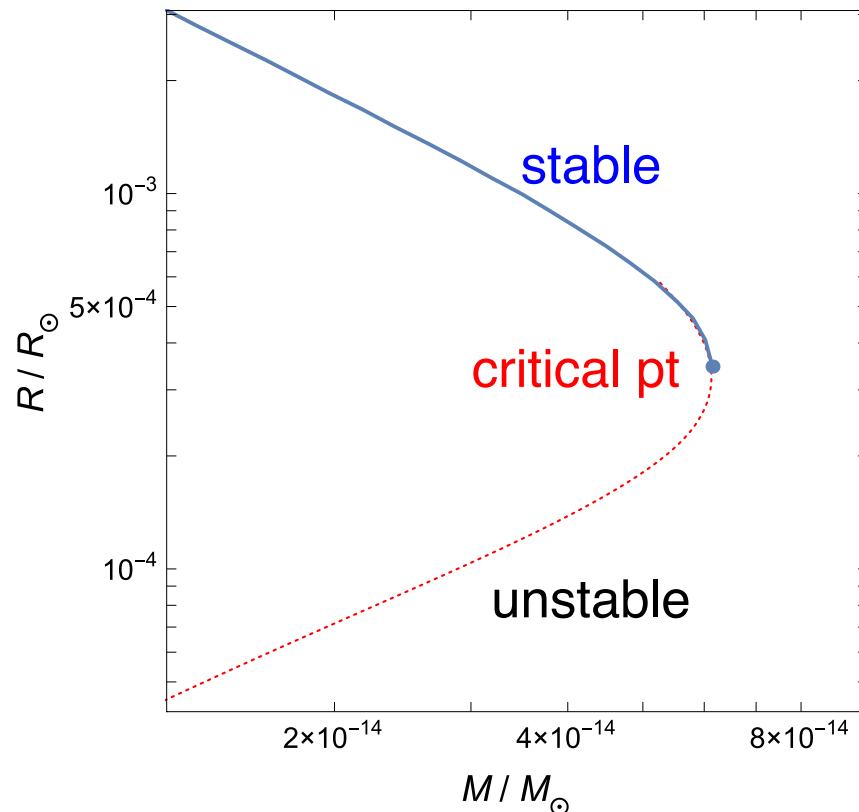
$$-\frac{\nabla^2}{2m_a}\psi + [(\mathcal{V}'_{\text{eff}}(\psi^*\psi) + m_a\Phi)]\psi = (\mu - m_a)\psi,$$

$$\nabla^2\Phi = 4\pi Gm_a\psi^*\psi.$$



(First) Critical Point

- Heavier dilute axion stars have smaller radii.
- **Critical mass:** beyond which the kinetic pressure cannot balance the attractive self-interaction and gravity



Critical point:

$$M_* = 10.2 f_a / \sqrt{G m_a^2}$$
$$= 6 \times 10^{-14} M_{\odot}$$

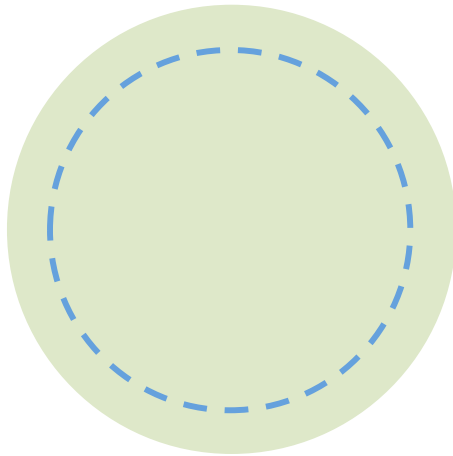
$$R_* = 3 \times 10^{-4} R_{\odot}$$
$$= 200 \text{ km}$$

Formation of Dilute Axion Stars

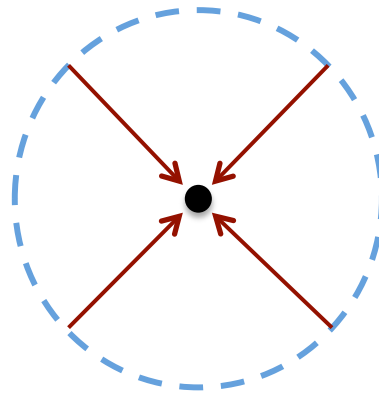
- Dilute axion stars can be produced in early universe.
- **Vacuum misalignment mechanism** produces coherent and non-relativistic axions.
- Spatial fluctuations in the axion field evolve into gravitationally bound “**miniclusters**” of axions.
- **Gravitational thermalization** drives the axion minicluster to form a dilute axion star.
- Dilute axion stars attract more axions and gradually reaches **the critical mass**.

End of Dilute Axion Stars

- Dilute axion stars collapse when its mass exceeds the critical mass,
- **What is the remnant?**

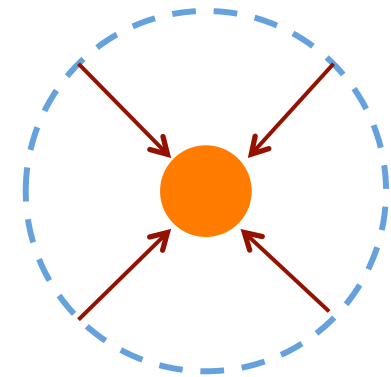


Less massive
dilute axion star?
by emitting
extra axions



Black hole: Schwarzschild
radius is ~ 20 orders of
magnitude smaller

Chavanis arXiv: 1604.05904
Helfer et al. arXiv: 1609.04724



Dense axion star?
Radius is 5 orders
smaller

Eby et al, arXiv: 1608.06911

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PRL 117, 121801 (2016)

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PRD 94, 076004 (2016)

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$$\mathcal{L}_{\text{eff}} = \frac{1}{2}i \left(\psi^* \dot{\psi} - \dot{\psi}^* \psi \right) - \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi - \mathcal{V}_{\text{eff}}$$

- Dilute axion field

$$\mathcal{V}_{\text{eff}} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \cancel{\frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4}} + \dots \quad \text{Dilute limit}$$

- In dense axion field $(\psi^* \psi) \sim m_a f_a^2$, must keep **all orders**

Both instanton and chiral potential can be summed to all orders

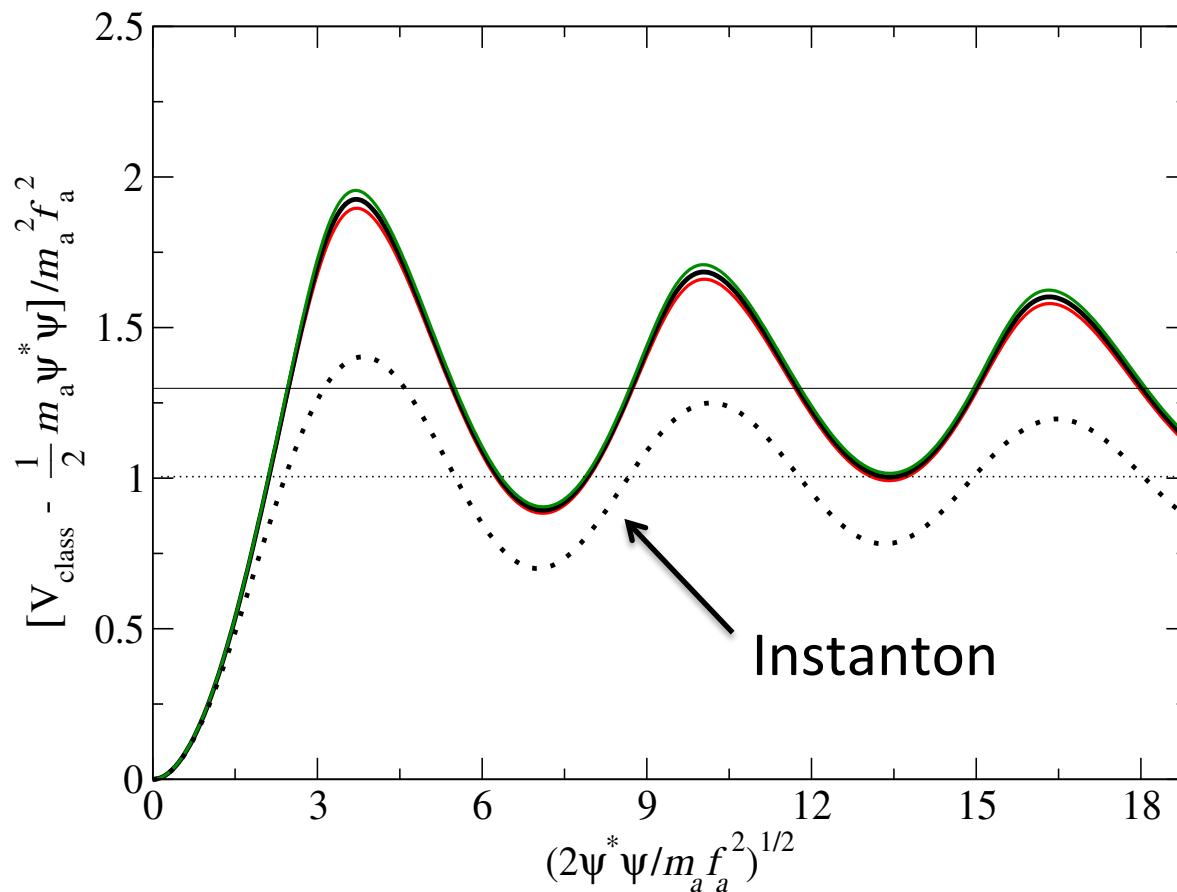
Instanton potential:

$$\mathcal{V}_{\text{eff}}(\psi^* \psi) = \frac{1}{2} m_a \psi^* \psi + m_a^2 f_a^2 \left[1 - J_0(2\psi^* \psi / m_a f_a^2) \right]$$

Eby, Suranyi, Vaz & Wijewardhana (2015)

Non-relativistic Instanton Potential

$$\mathcal{V}_{\text{eff}}(\psi^* \psi) = \frac{1}{2} m_a \psi^* \psi + m_a^2 f_a^2 \left[1 - J_0(2\psi^* \psi / m_a f_a^2) \right]$$



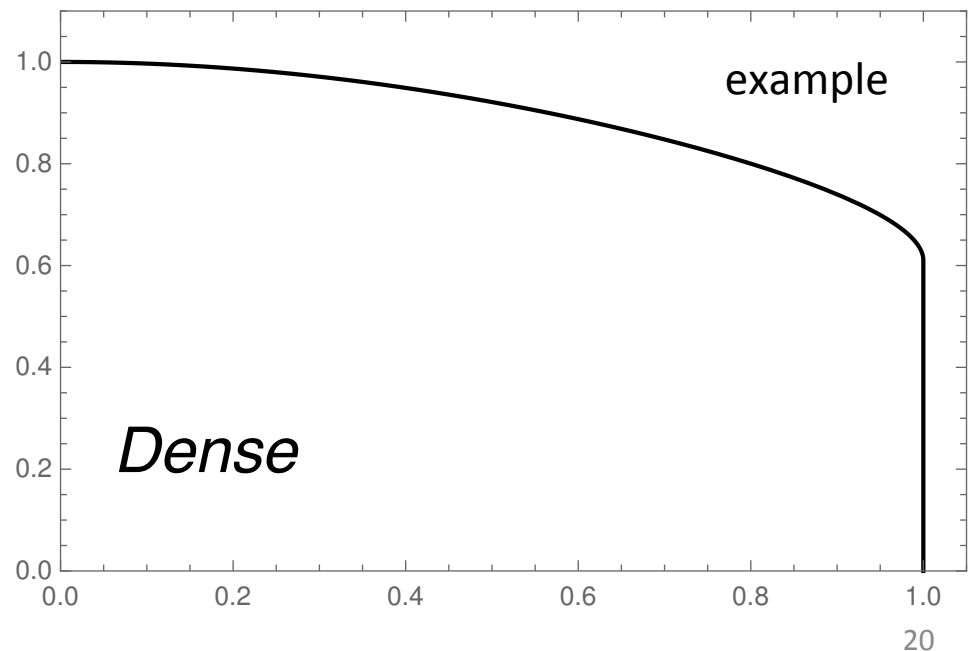
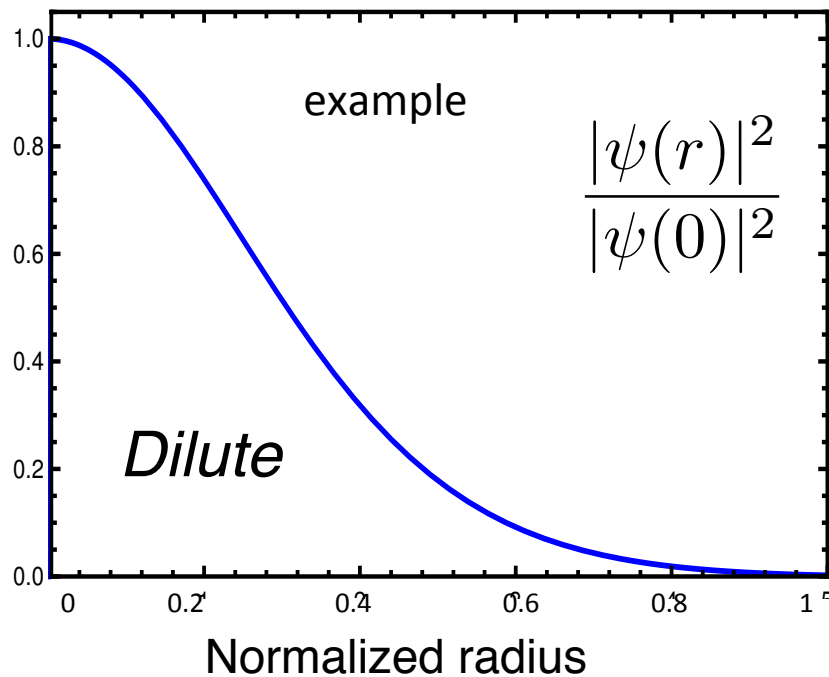
Not periodic
Decreasing Amplitude

Dense Axion Stars

- Assume:
- Instanton potential
 - Newtonian gravity
 - Spherically symmetric

Compare axion number density

- Dilute axion star: **Gaussian-like**
- Dense axion star: **almost flat, with a fast-dropping edge**

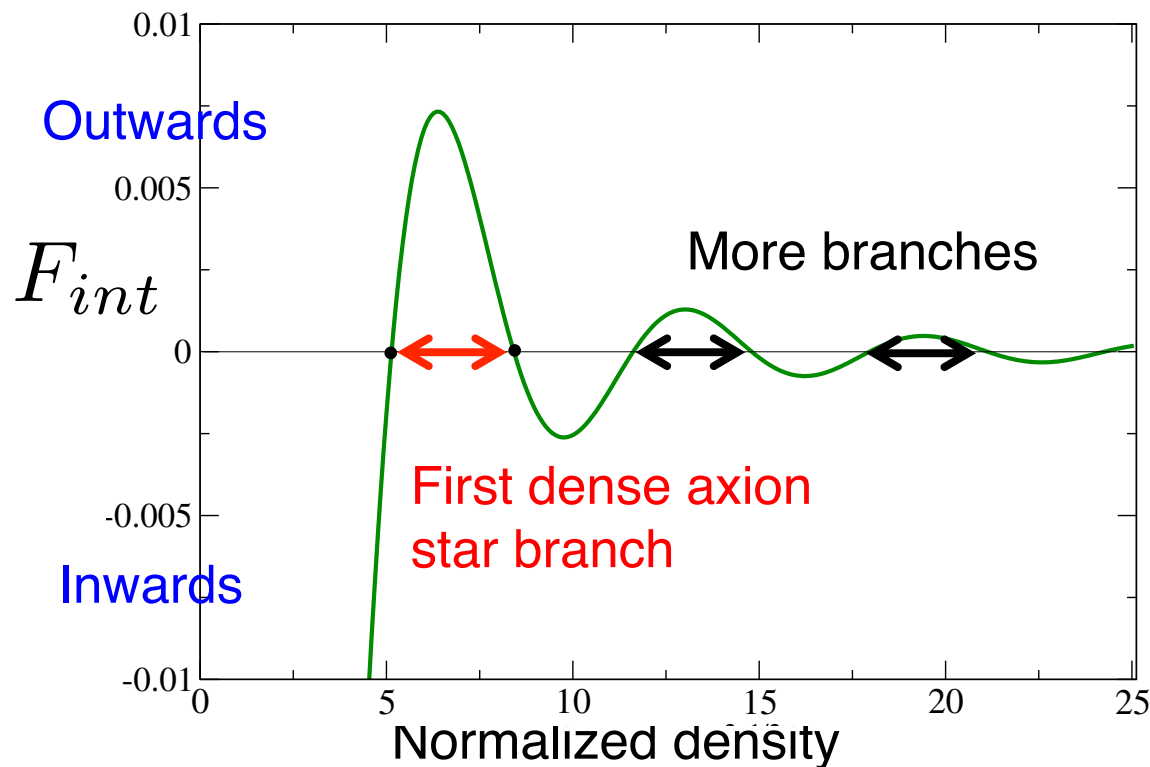


Braaten, Mohapatra, HZ, PRL (2016)

Self-interaction Force

$$-\frac{\nabla^2}{2m_a}\psi + [(\mathcal{V}'_{\text{eff}}(\psi^*\psi) + m_a\Phi)]\psi = (\mu - m_a)\psi, \quad \nabla^2\Phi = 4\pi Gm_a\psi^*\psi.$$

- Self-interaction force (mean-field pressure) $F_{int} = -\mathcal{V}''_{\text{eff}}(\psi^*\psi) \underbrace{\frac{d}{dr}(\psi^*\psi)}_{\text{Smaller than 0}}$

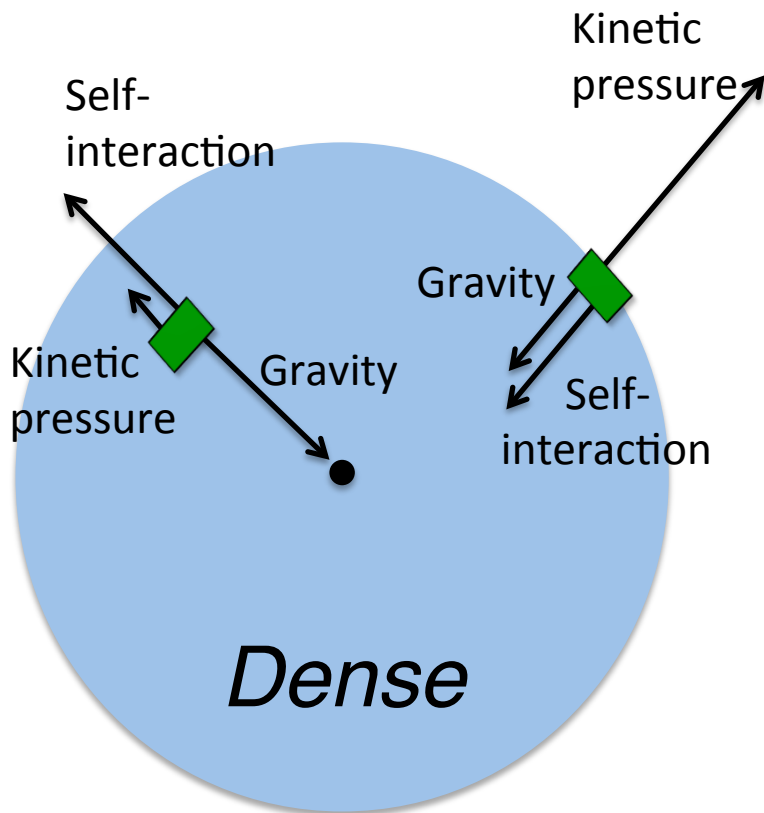


Smaller than 0

Forces Balancing

$$-\frac{\nabla^2}{2m_a}\psi + [(\mathcal{V}'_{\text{eff}}(\psi^*\psi) + m_a\Phi)]\psi = (\mu - m_a)\psi, \quad \nabla^2\Phi = 4\pi Gm_a\psi^*\psi.$$

- Recall in dilute axion star, kinetic pressure balances gravity and self-interaction force



- In dense axion star

Bulk:

- self-interaction force balances gravity,
- kinetic pressure ~ 0
- wave-function is almost flat

Surface:

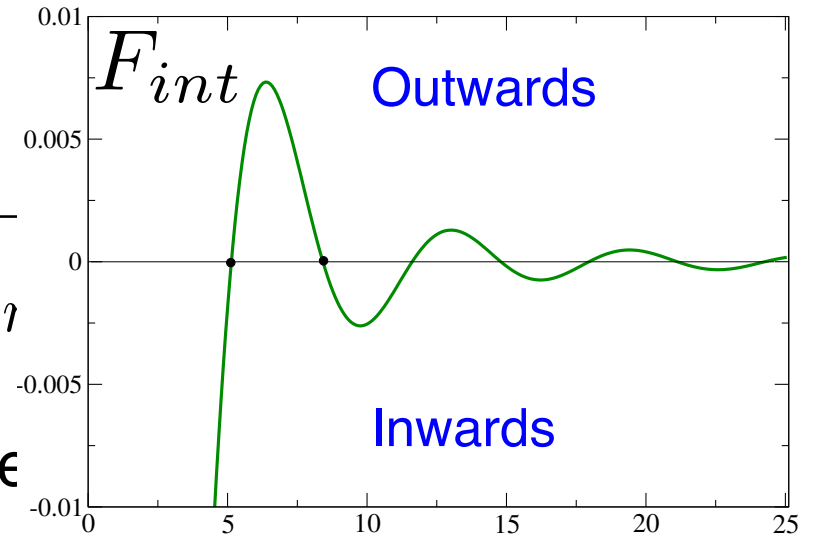
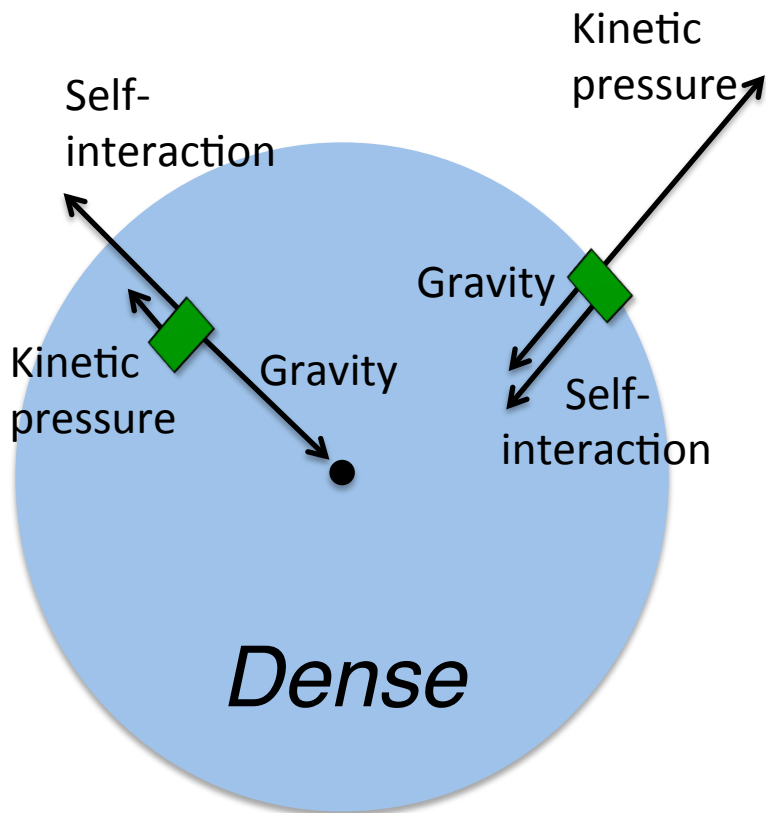
- large kinetic pressure needed to balance the other two,
- wave-function drop rapidly

Braaten, Mohapatra, HZ, PRL (2016)

Forces Balancing

$$-\frac{\nabla^2}{2m_a} \psi + [(\mathcal{V}'_{\text{eff}}(\psi^* \psi) + m_a \Phi)] \psi = (\mu - \gamma)$$

- Recall in dilute axion star, kinetic pressure and self-interaction force



- In dense axion star

Bulk:

- self-interaction force balances gravity,
- kinetic pressure ~ 0
- wave-function is almost flat

Surface:

- large kinetic pressure needed to balance the other two,
- wave-function drops rapidly

Braaten, Mohapatra, HZ, PRL (2016)

Thomas-Fermi Approximation

- When the surface thickness is small compare to the bulk, Thomas-Fermi approximation can be applied.

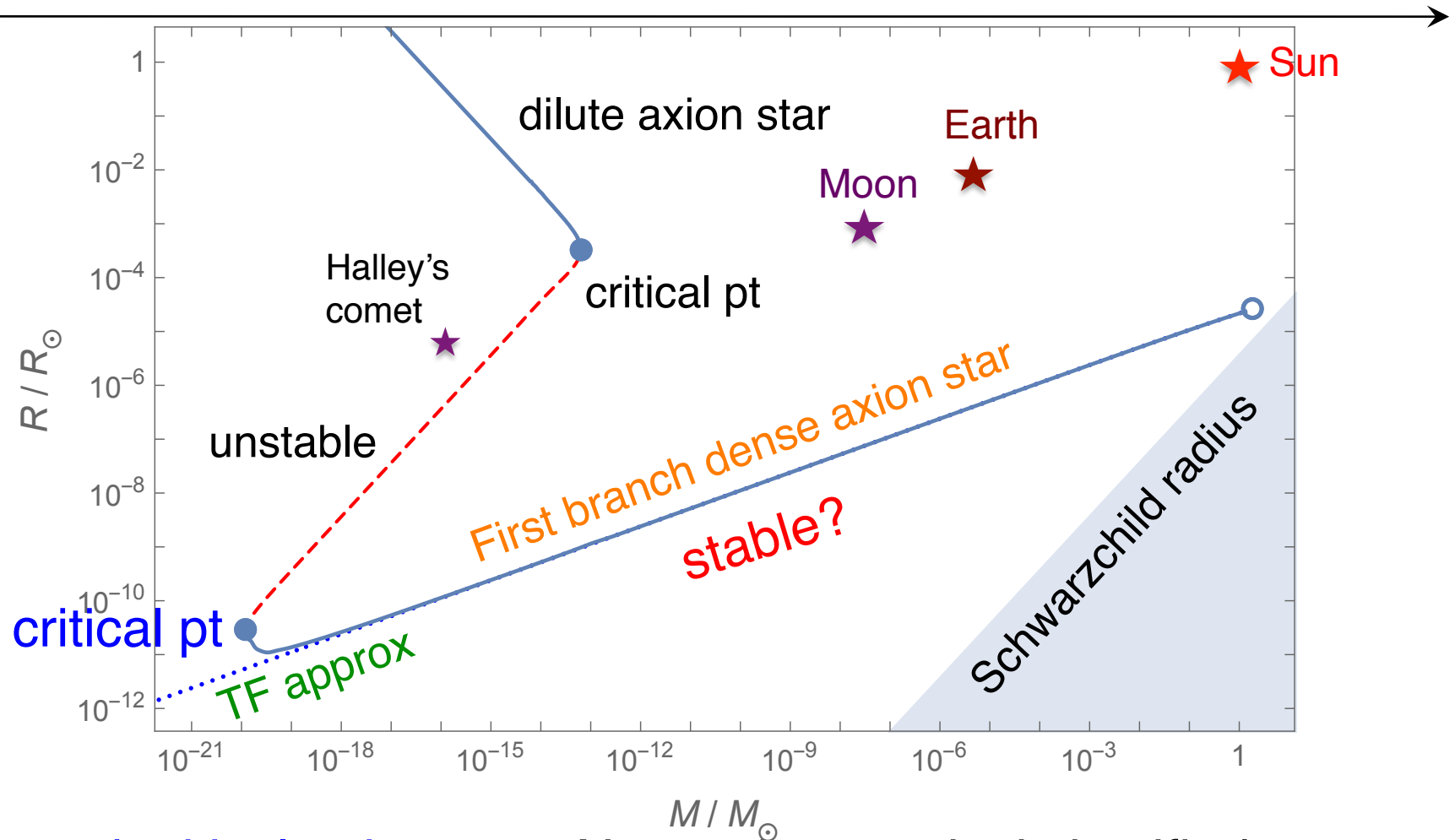
$$-\cancel{\frac{\nabla^2}{2m_a}}\psi + [(\mathcal{V}'_{\text{eff}}(\psi^*\psi) + m_a\Phi)]\psi = (\mu - m_a)\psi,$$

$$\nabla^2\Phi = 4\pi Gm_a\psi^*\psi.$$

Greatly simplifies!

- **Interaction force** (mean-field pressure) exactly balances **gravitational force**
- Not applicable to small dense axion star, in which the surface thickness is important

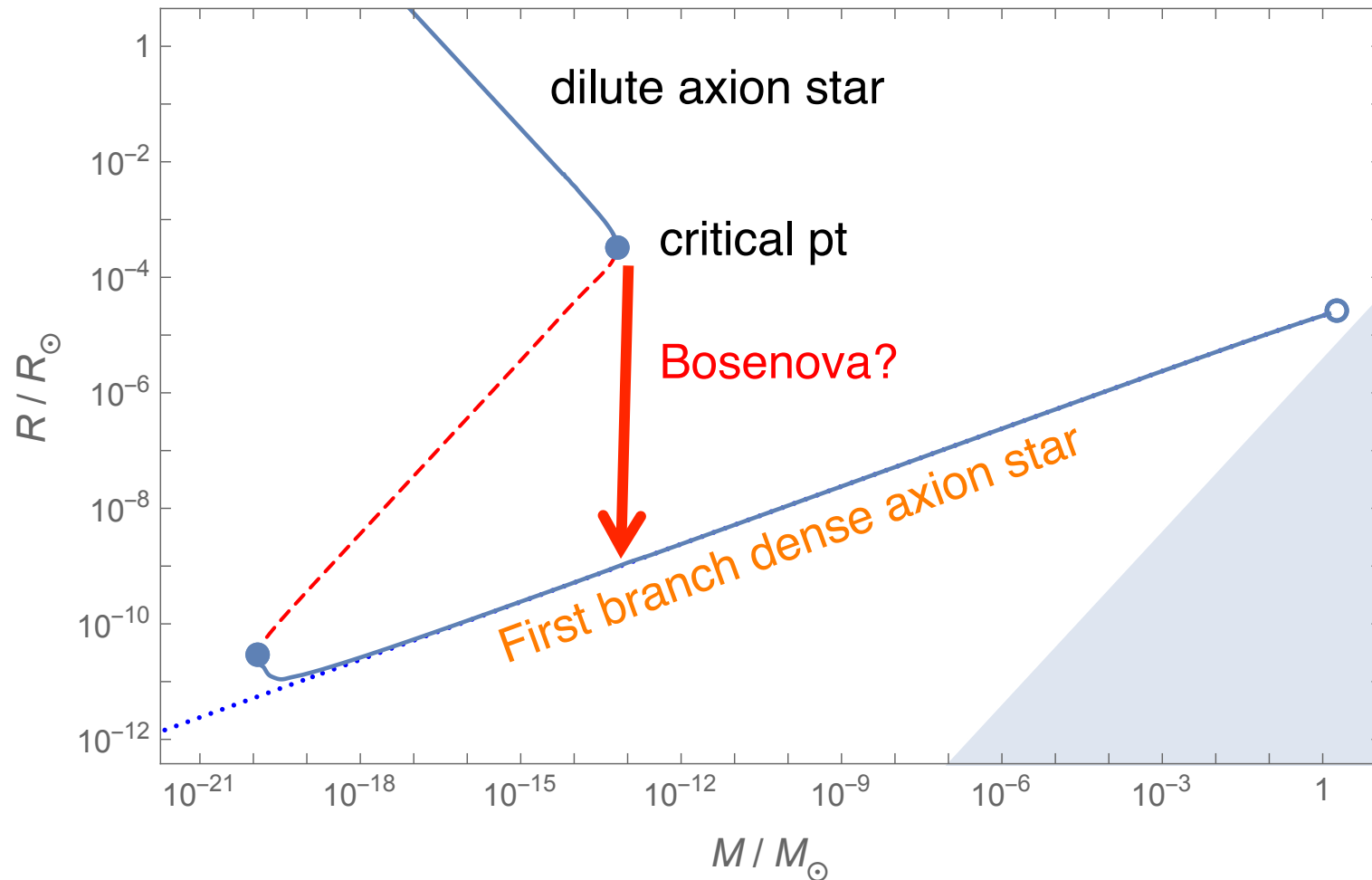
Radius vs. Mass



- 2nd critical point
- Newtonian gravity is justified
- Heavier dense axion stars have larger radii

Formation of Dense Axion Stars

- Possible remnants of dilute axion stars collapsing



Braaten, Mohapatra, HZ, PRL (2016)

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PRL 117, 121801 (2016)

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arXiv:1609.05182

✧ Axion EFT

PRD 94, 076004 (2016)

✧ Summary

Axion Detection

- Depends on the tiny axion-photon coupling

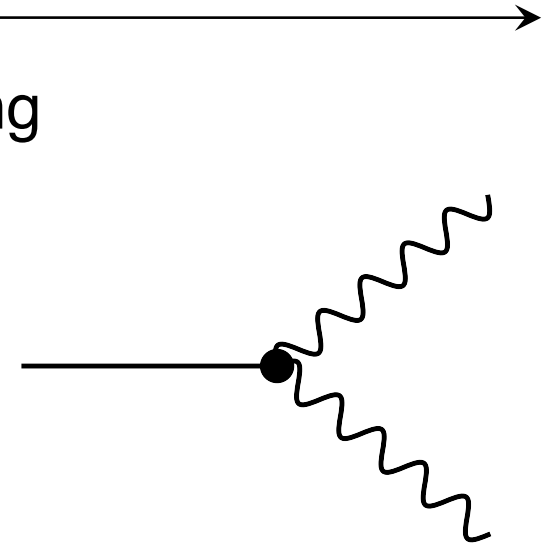
$$\mathcal{L}_{\text{em}} = \frac{c_{\text{em}} \alpha}{16\pi f_a} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \phi.$$

$$c_{\text{em}} \sim 1 \quad \text{Model dependent}$$

Suppressed by $f_a \sim 10^{11} \text{ GeV}$

$$\Gamma_a = \frac{c_{\text{em}}^2 \alpha^2 m_a^3}{256\pi^3 f_a^2}.$$

Axion lifetime $\sim 10^{36}$ years
Age of Universe $\sim 10^{10}$ years



- Direct detection, **indirect detection** and laser experiment.

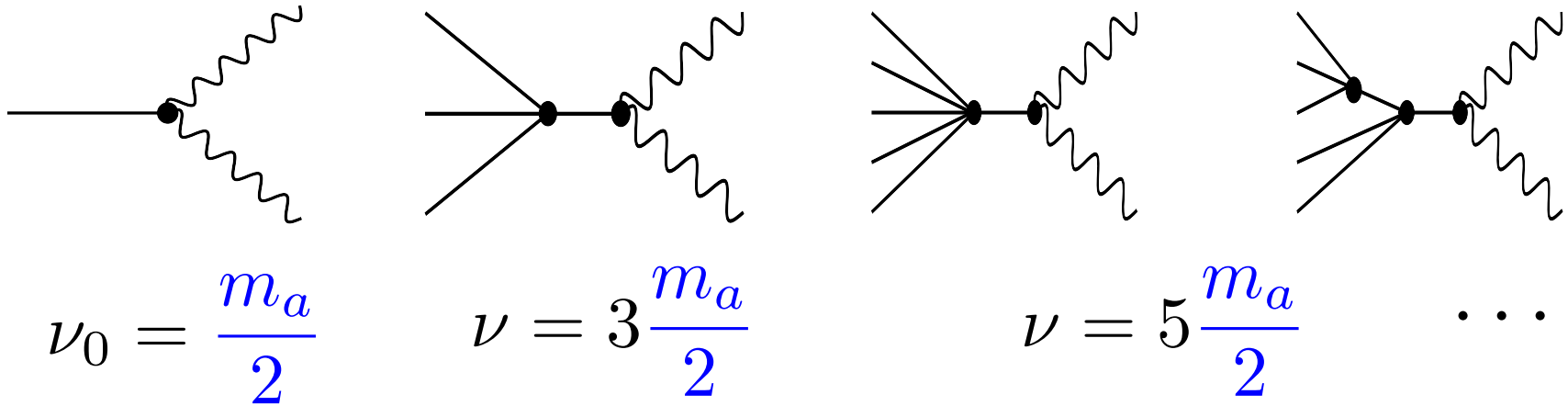
Lect. Notes Phys. 741 (2008)

A recent review: Kim & Carosi (2010)

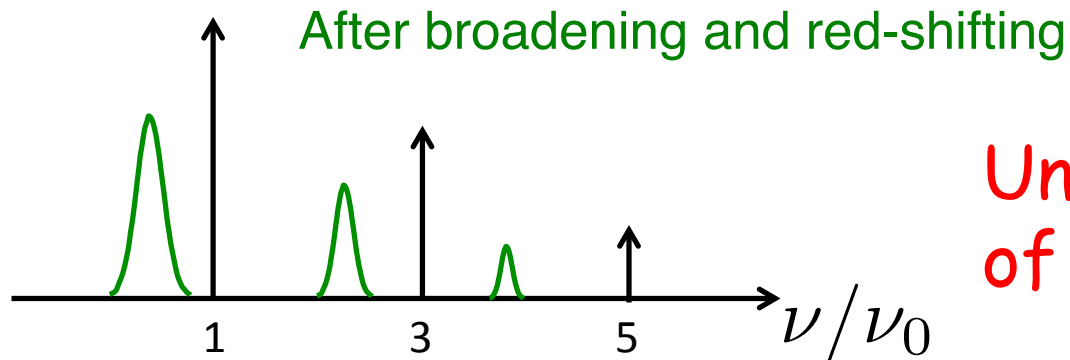
Indirect Detection

- Detect **radio-frequency** photons
- Currently no experiment available (tiny $a\gamma\gamma$ coupling)
- **Two photon-production channels**
 - One-step: NR axions $\rightarrow \gamma$
 - Two-step: NR axions \rightarrow relativistic axions $\rightarrow \gamma$

1st Channel: NR Axions to Photons



- **Odd-integer harmonics** of the **fundamental radio frequency**.

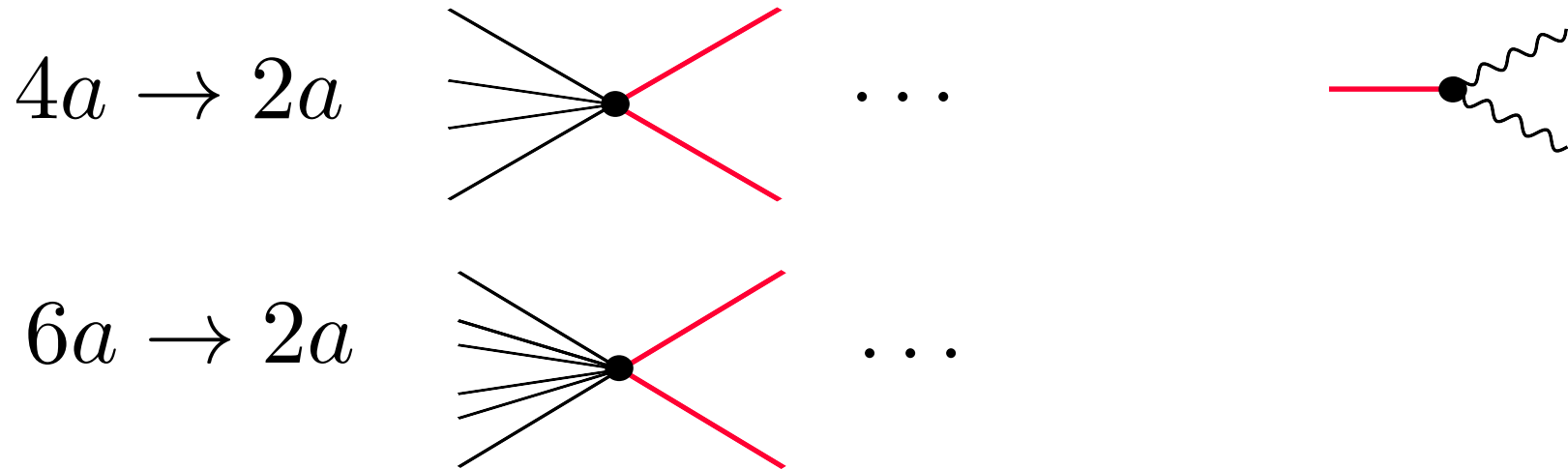


**Unique feature
of axions !!**

2nd Channel

- Two relativistic axions production

suppressed by one power of $(m_a^2/f_a^2) \sim 10^{-48}$

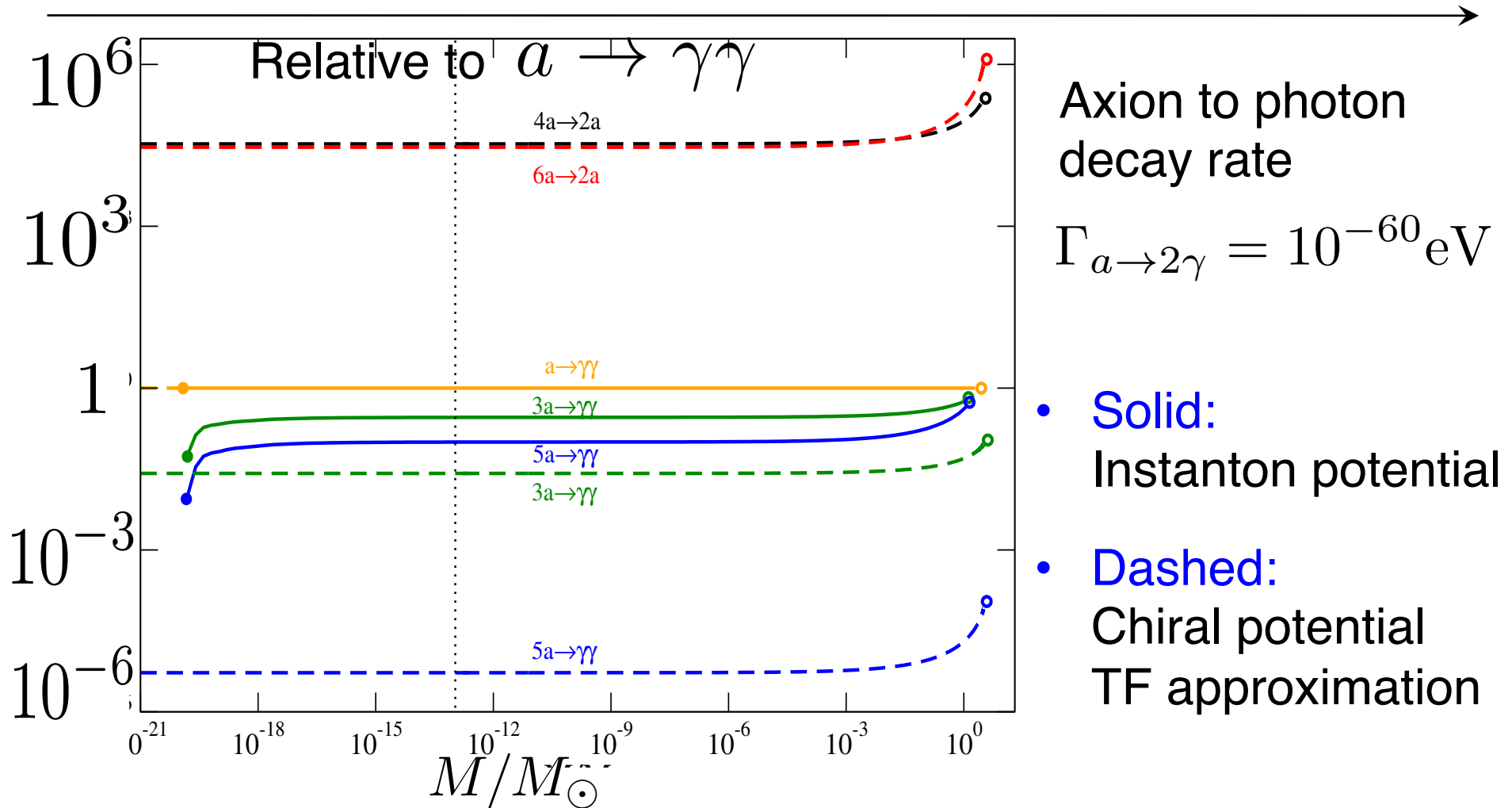


- More relativistic axions suppressed by more powers
- Much weaker photon signal than 1st channel

Two Types of Sources

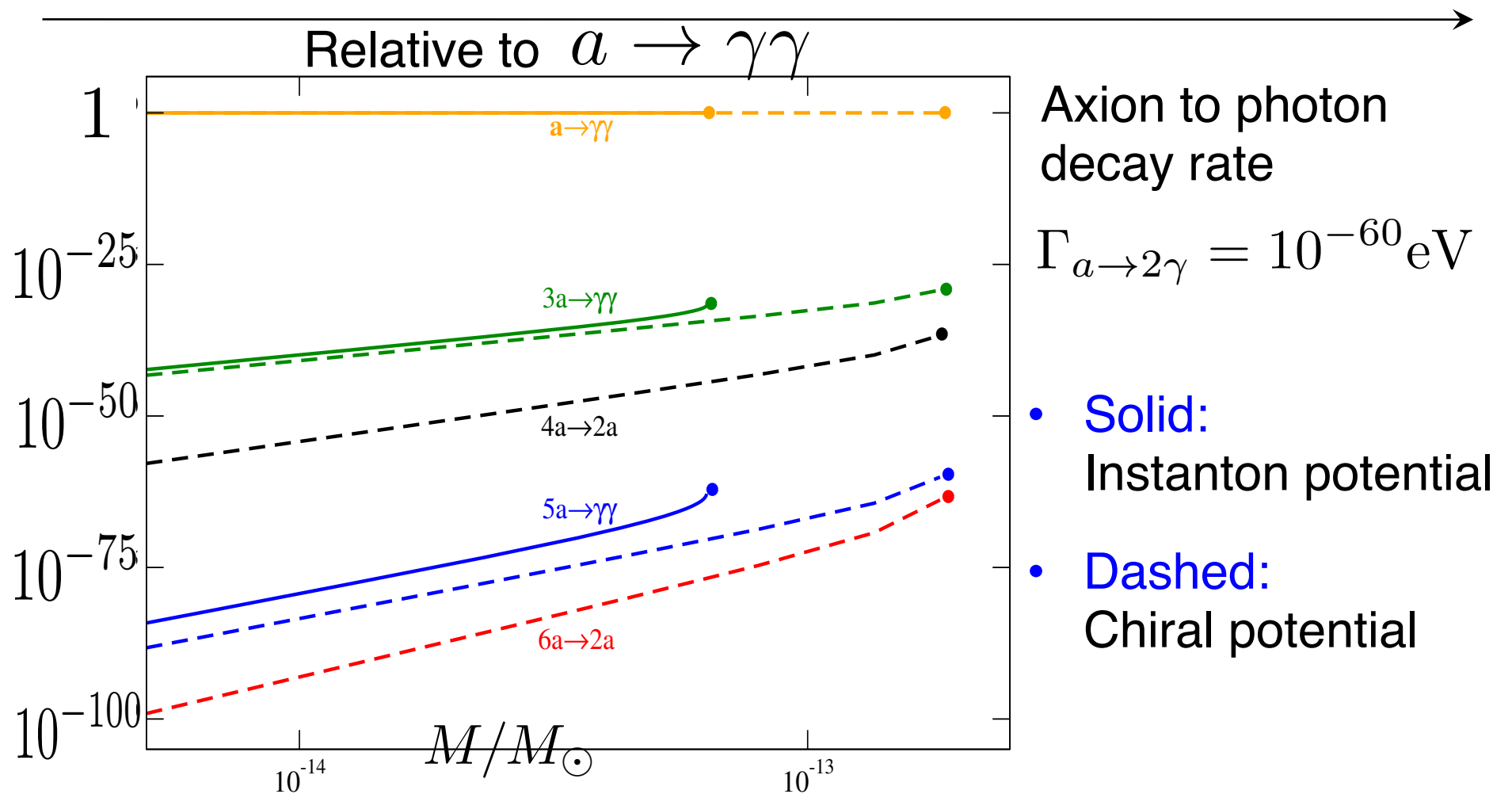
- Continuous photon emission
 - Stable axion stars
- Catastrophic phenomenon:
 - a lot of energy released in a short time
 - Collapse of dilute Axion stars
 - Collision of an axion star with a neutron star
 - ...

Emission From **Dense** Axion Star



$4a \rightarrow 2a, 6a \rightarrow 2a, 8a \rightarrow 2a$ are all zero for instanton potential

Emission From Dilute Axion Star



Axion to photon
decay rate

$$\Gamma_{a \rightarrow 2\gamma} = 10^{-60} \text{eV}$$

$4a \rightarrow 2a, 6a \rightarrow 2a, 8a \rightarrow 2a$ are all zero for instanton potential

Photon Flux Estimate

- Single axion decay to two photons is independent of the configuration.

- Solar system

radius $\sim 125,000$ AU, DM density $\sim 0.3 \text{ GeV/cm}^3$

Total DM in Solar system $\sim 0.01 M_{\odot}$ 10^{68} axions

Photon production rate $\sim 10^{22}/\text{sec}$

Energy released: $\sim 10^9 \text{ GeV/sec} \sim 0.4$ Watt

- Milky way:

Total DM: $\sim 10^{12} M_{\odot}$ 10^{82} axions

Largest 1st branch dense axion star has $\sim 10^{70}$ axions
 $\sim 10^{11} \text{ GeV/sec} \sim 40$ Watt

Axion Stars are Not So Bright !

PRL 117, 121801 (2016) Editors' suggestion



Picture chosen by PRL

Hydrogen Axion Star ?

- Hydrogen gas is captured by the gravitational potential well of axion star, forming **dense metallic fluid** state.
- Electron interacts with axion, generating heat, resulting in **blackbody radiation** with peak in the **UV** region.
- Energy released: $10^{13} \text{ W} \times \left(\frac{m_a}{5 \text{ meV}} \right)^4$
- The signal should be readily visible to current high-resolution telescopes.

Bai, Barger and Berger, JHEP (2016)

Catastrophic Phenomena

Fast Radio Burst (FRB)

- A ultra-fast (**milli-sec**) burst of photons in **radio frequency**.
- Nothing similar observed in optical, X rays and γ rays
- Since 2007, 17 events have been reported.
- Estimated rate $\sim 10^4 \text{ sky}^{-1} \text{ day}^{-1}$
- Reported frequency is **1.4 GHz** (telescope design)
- **Extra-galactic** sources from dispersion measure
- Energy released up to 10^{40} erg $\sim 10^{-14} M_{\odot}$ (If isotropic)
- **Strong linear polarization** observed.

Recent review: Katz, arXiv:1604.01799

Online database: <http://www.astronomy.swin.edu.au/pulsar/frbcats>

Fast Radio Burst

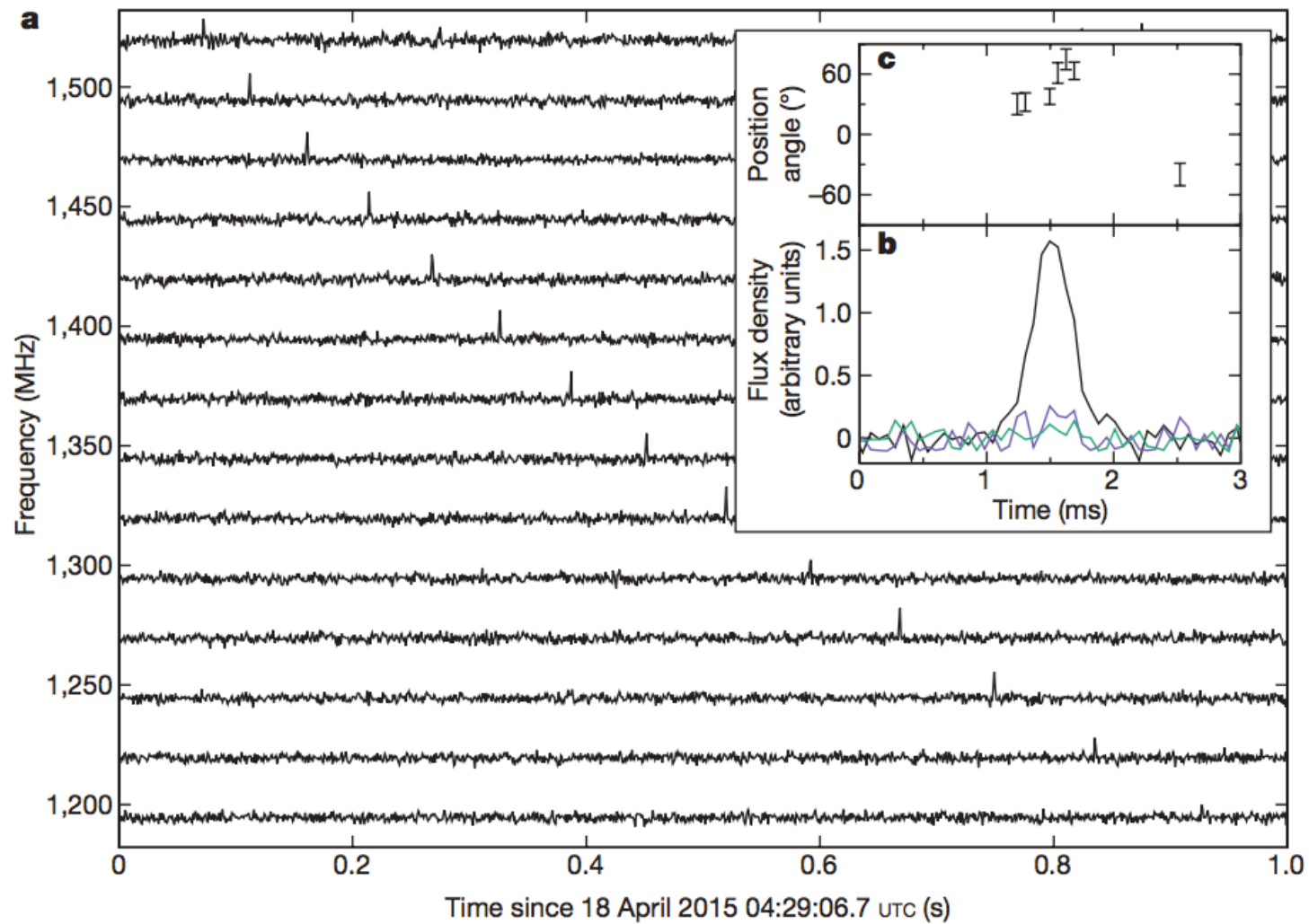


Figure from Nature 530, 453 (2016)

Are Axion Stars an Explanation?

- ✓ Observed frequency: 1.4 GHz

$$10^{-6} \text{eV} < m_a < 10^{-2} \text{eV} \quad 0.2 \text{ GHz} < \nu < 2400 \text{ GHz}$$

Also explains why such burst is not observed in other bands.

- ✓ Total energy released: up to $\sim 10^{-14} M_{\odot}$

- Dilute axion star critical mass $6 \times 10^{-14} M_{\odot}$
- Dense axion star mass $10^{-20} M_{\odot}$ to $2 M_{\odot}$

- ✓ Time duration: $\sim 1 \text{ ms}$

- Dilute axion star critical radius: 200 km
- Dense axion star radius: 1m to 10 km

- ✓ Polarized photons

Axions in axion stars are in coherence

Scenarios with Axion Stars

- Collision of a dilute axion star with a neutron star

Coherent electric dipole radiation

- From electrons in atmosphere Iwazaki, hep-ph/9908468
- From neutrons in outer core Raby, PRD (2016)

- Collapse of dilute axion stars above the critical mass

Tkachev, JETP Lett. (2014)

- Collision of two axion stars

Eby et.al., arXiv:1701.01476

- Collision of a dense axion star with a neutron star

Observe Odd-integer Harmonics

- One **unique** feature of axion stars:
odd-integer harmonics of the **fundamental radio frequency**.
- Can we observe the fast radio burst at other frequencies?
 1.4 GHz , $3 \times 1.4 \text{ GHz}$, $5 \times 1.4 \text{ GHz} \dots$
or
 $1/3 \times 1.4 \text{ GHz}$, 1.4 GHz , $5/3 \times 1.4 \text{ GHz} \dots$
Many possible combinations
- Need more events in more frequency windows.

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- **Non-relativistic axions:** complex scalar

$$\mathcal{H}_{\text{eff}} = \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi + \mathcal{V}_{\text{eff}}(\psi^* \psi)$$

- Integrate out axion mass scale
 - Much simpler, but equally accurate in the NR limit
- **Need to find the NR potential $\mathcal{V}_{\text{eff}}(\psi^* \psi)$**

Naïve NR Reduction

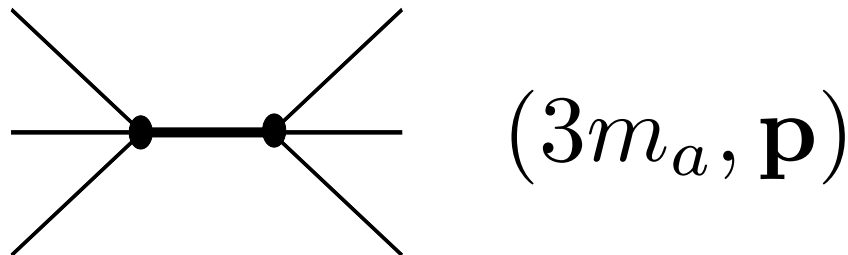
- Zeroth order approximation

$$\phi(\mathbf{r}, t) = \frac{1}{\sqrt{2m_a}} [\psi(\mathbf{r}, t)e^{-im_a t} + \psi^*(\mathbf{r}, t)e^{+im_a t}]$$

Dilute $\mathcal{V}_{\text{eff}} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4} + \dots$

Dense $\mathcal{V}_{\text{eff}}(\psi^* \psi) = \frac{1}{2} m_a \psi^* \psi + m_a^2 f_a^2 [1 - J_0(2\psi^* \psi / m_a f_a^2)]$

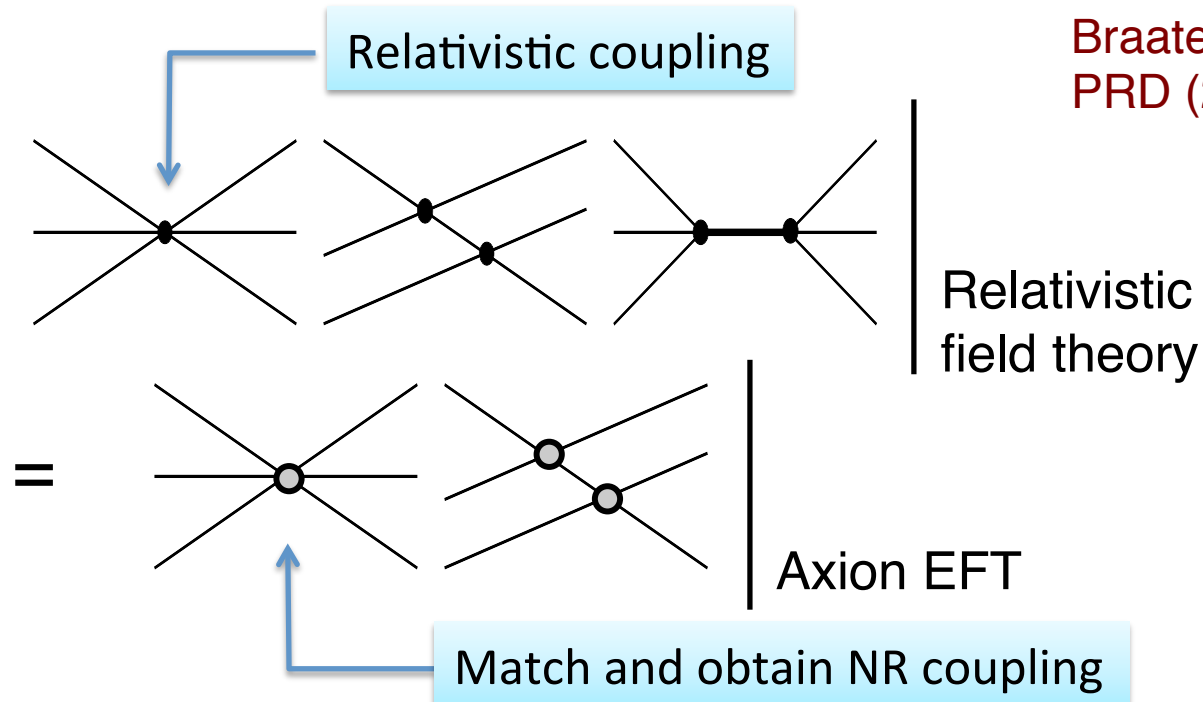
- Used to get dilute and dense axion star solutions
- Not considering **virtual axions**



Braaten, Mohapatra, HZ, PRD (2016)

Match the amplitude

- Matching **low-energy scattering** amplitudes.
- Includes **all** virtual axion contributions
- **Only tree diagram**: loops are suppressed by $(m_a/f_a)^2 \sim 10^{-48}$
- Example: 3 to 3 scattering



Match Low-power Couplings

- Expand the NR potential

$$\mathcal{V}_{\text{eff}}(\psi^* \psi) = m_a \psi^* \psi + m_a^2 f_a^2 \sum_{n=2}^{\infty} \frac{v_n}{(n!)^2} \left(\frac{\psi^* \psi}{2m_a f_a^2} \right)^n .$$

- Check (v_2, v_3, v_4, v_5) for instanton potential

NR reduction: $(-1, 1, -1, 1)$

With matching: $(-1, -1.125, -2.25, 1.76)$

Deviation: $(0, -189\%, -56\%, -43\%)$

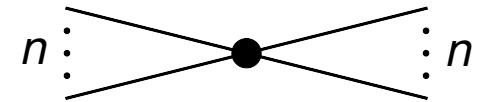
Contribution of virtual axions is important !

Dense Regime

- Cannot truncate the power expansion
- Impossible to extract all couplings by matching (infinitely many)
- One scheme: include more and more virtual axion propagators in the matching

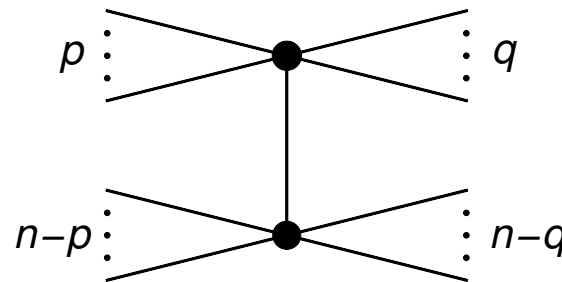
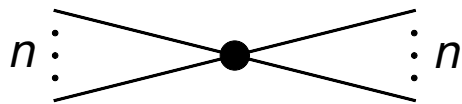
Naïve NR reduction

Match diagrams with **no** virtual propagator



1st improvement

Match diagrams with **0 or 1** virtual propagator



Braaten, Mohapatra, HZ, PRD (2016)

Summary

- Gravity can thermalize axions toward **Bose-Einstein condensates** and form **dilute axion stars**.
- A dilute axion star accumulates axions and collapses once its mass exceeds the critical mass $10^{-14} M_{\odot}$
- **Dense axion star** is a possible remnant.
- Catastrophic phenomena involving axion stars can release **a large amount** of **coherent radio-frequency** photons in a **very short time**, which may explain **fast radio burst**.
- The photons in **odd-integer harmonics** of a fundamental radio frequency are a unique signature of axions.