Modification of Higgs Couplings in Minimal Composite Models

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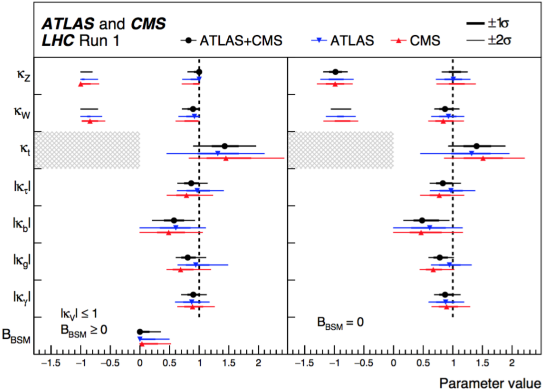
With Ian Low, Carlos E. M. Wagner, arXiv:1703.xxxx

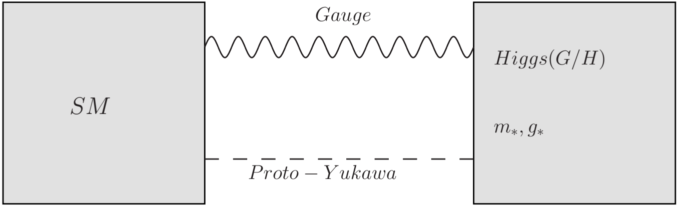
Motivation



LHC data prefer $|\kappa_t| > |\kappa_g|$!

Composite Higgs model scenario

- Two sectors: the elementary sector, the composite (strong) sector.
- Higgs are pseudo-Goldstone bosons living in some coset G/H
- SM fermions acquire masses from linear mixing.



General Analysis: Higgs are Goldstone bosons G/H

The whole strong dynamics encoded in the form factors:

$$\sum_{q=t,b} \Pi_{q_L}\ \bar{q}_L \not{p}\ q_L + \Pi_{q_R}\ \bar{q}_R \not{p}\ q_R - (\Pi_{q_L q_R}\ \bar{q}_L\ q_R + \mathrm{h.c.})$$

$\Downarrow$ Compact coset

$$\Pi_{q_L} = \Pi_{0q_L} + s_h^2\ \Pi_{1q_L} + s_h^4\ \Pi_{2q_L} + \cdots\ ,$$
$$\Pi_{q_R} = \Pi_{0q_R} + s_h^2\ \Pi_{1q_R} + s_h^4\ \Pi_{2q_R} + \cdots\ ,$$

$\Downarrow$ vectorial representations

$$\Pi_{q_L q_R} = s_h c_h \left(\Pi_{1q_L q_R} + s_h^2\ \Pi_{2q_L q_R} + \cdots\right),$$

$$\boxed{s_h \equiv \sin h/f\,, c_h \equiv \cos h/f}$$

Partial compositeness

Partial compositeness tells us:

$$\mathcal{L}_{mix}^{UV} = (\bar{q}_L)_\alpha (y_L)^\alpha{}_I \mathcal{O}^I_{q_L} + \bar{t}_R (y^t_R)_I \mathcal{O}^I_{t_R} + \bar{b}_R (y^b_R)_I \mathcal{O}^I_{b_R}$$

$\Downarrow$ IR

$$-\mathcal{L}_m = (\bar{F}_L, \bar{\vec{\Psi}}_L) M_F(h) \begin{pmatrix} F_R \\ \vec{\Psi}_R \end{pmatrix}, \qquad M_F = \begin{pmatrix} 0 & Y_L^T(h) \\ Y_R(h) & M_c \end{pmatrix},$$

$\Downarrow$ E.O.M $\qquad\qquad$ $\Downarrow$

$$\Pi_{f_L f_R}(0) = -Y_L^T M_c^{-1} Y_R, \qquad \text{Det}\, M_F = -Y_L^T M_c^{-1} Y_R \ \text{Det}\, M_c,$$

$\Downarrow$

$$\boxed{\text{Det}\, M_F = \Pi_{f_L f_R}(0)\ \text{Det}\, M_c}$$

Higgs couplings from the form factors

The quark masses are evaluated at the zero momentum:

$$m_q = \frac{\Pi_{q_L q_R}(0)}{\sqrt{\Pi_{q_L}(0)}\sqrt{\Pi_{q_R}(0)}}$$

$$\Downarrow \; v\frac{\partial}{\partial \langle h \rangle} = \sin\theta \frac{\partial}{\partial \theta}$$

$$\begin{aligned} c_q &\equiv \frac{g_{hq\bar{q}}}{(g_{hq\bar{q}})_{\mathrm{SM}}} = \frac{v}{m_q}\frac{\partial m_q}{\partial \langle h \rangle} = \sin\theta \frac{\partial}{\partial \theta} \log m_q \\ &= \sin\theta \frac{\partial}{\partial \theta} \log \Pi_{q_L q_R} - \frac{1}{2}\sin\theta \frac{\partial}{\partial \theta}\left(\log \Pi_{q_L} + \log \Pi_{q_R}\right) \end{aligned}$$

$$\boxed{\theta = \langle h \rangle / f, \quad v = f \sin\theta = 246\,\mathrm{GeV}}$$

Higgs couplings from the form factors

Due to partial compositeness, the *ggh* coupling can be obtained by the form factors:

$$c_g = c_g^{(t)} + c_g^{(b)}$$

$$c_g^{(t)} \equiv \frac{g_{\rm ggh}^{(t)}}{(g_{\rm ggh})_{\sf SM}} = \sin\theta \frac{\partial}{\partial\theta} \log \, {\rm Det} M_{2/3} = \sin\theta \frac{\partial}{\partial\theta} \log \Pi_{t_L t_R}$$

$$c_g^{(b)} \equiv \frac{g_{\rm ggh}^{(b)}}{(g_{\rm ggh})_{\sf SM}} = \sin\theta \frac{\partial}{\partial\theta} \left(\log \Pi_{b_L b_R} - \log m_b\right)$$

$$= \frac{1}{2} \sin\theta \frac{\partial}{\partial\theta} \log \Pi_{b_L} + \frac{1}{2} \sin\theta \frac{\partial}{\partial\theta} \log \Pi_{b_R}$$

We have:

$$c_t - c_g = -\frac{1}{2} \sin\theta \frac{\partial}{\partial\theta} \left(\log \Pi_{t_L} + \log \Pi_{t_R} + \log \Pi_{b_L} + \log \Pi_{b_R}\right)$$

The coupling difference $c_t - c_g$

The coupling difference is controlled by wave function normalization:

$$c_t - c_g = -\frac{1}{2}\sin\theta\frac{\partial}{\partial\theta}\left(\log\Pi_{t_L} + \log\Pi_{t_R} + \log\Pi_{b_L} + \log\Pi_{b_R}\right)$$

Recall the expansion:

$$\Pi_{q_L} = \Pi_{0q_L} + s_h^2\ \Pi_{1q_L} + s_h^4\ \Pi_{2q_L} + \cdots\ ,$$
$$\Pi_{q_R} = \Pi_{0q_R} + s_h^2\ \Pi_{1q_R} + s_h^4\ \Pi_{2q_R} + \cdots\ ,$$

$$\Downarrow \xi = \sin^2\theta \ll 1$$

$$\boxed{c_t - c_g = -\xi\left(\frac{\Pi_{1t_L}}{\Pi_{0t_L}} + \frac{\Pi_{1t_R}}{\Pi_{0t_R}} + \frac{\Pi_{1b_L}}{\Pi_{0b_L}} + \frac{\Pi_{1b_R}}{\Pi_{0b_R}}\right) + \cdots}$$

Higgs potential from the form factors

The Coleman-Weinberg potential for the Higgs boson:

$$V_f(h) = -2N_c \int \frac{d^4Q}{(2\pi)^4} \left[\log\left(Q^2\ \Pi_{t_L}\Pi_{t_R} + |\Pi_{t_L t_R}|^2\right) + t \to b\right]$$

Expand in s_h:

$$V_f(h) \simeq -\gamma_f\ s_h^2 + \beta_f\ s_h^4$$

The leading contribution to the γ_f factor:

$$\boxed{\gamma_f^{y^2} = \frac{2N_c}{(4\pi)^2} \int_0^{\Lambda^2} dQ^2\ Q^2 \left[\frac{\Pi_{1t_L}}{\Pi_{0t_L}} + \frac{\Pi_{1t_R}}{\Pi_{0t_R}} + \frac{\Pi_{1b_L}}{\Pi_{0b_L}} + \frac{\Pi_{1b_R}}{\Pi_{0b_R}}\right]}$$

Relation between $c_t - c_g$ and Higgs mass term

They are related by the master function:

$$\boxed{\mathcal{F}(Q^2) = \frac{\Pi_{1t_L}}{\Pi_{0t_L}} + \frac{\Pi_{1b_L}}{\Pi_{0b_L}} + \frac{\Pi_{1t_R}}{\Pi_{0t_R}} + \frac{\Pi_{1b_R}}{\Pi_{0b_R}}}$$

$$c_t - c_g = -\mathcal{F}(0)\xi + \cdots\,, \qquad \gamma_f \sim \frac{2N_c}{(4\pi)^2}\int_0^{\Lambda^2} dQ^2\, Q^2 \mathcal{F}(Q^2)\ .$$

The slope of the integrand at the origin:

$$[x\mathcal{F}(x)]'|_{x=0} = (\mathcal{F}(x) + x\mathcal{F}'(x))|_{x=0} = \mathcal{F}(0)$$

Roughly, we have:

$$\boxed{\gamma_f > 0 \quad \Rightarrow \quad c_t < c_g}$$

Example: 5 of $SO(5)/SO(4)$

Neglecting the kinetic terms, the effective Lagrangian:

$$\mathcal{L}^{M4_5} = -M_4 \bar{\Psi}\Psi + \left[c_4 y_L f (\bar{q}_L^5)_I U^I{}_i \Psi_R^i + a_4 y_R f (\bar{t}_R^5)_I U^I{}_i \Psi_L^i + h.c. \right]$$

$$\mathcal{L}^{M1_5} = -M_1 \bar{\Psi}\Psi + \left[c_1 y_L f (\bar{q}_L^5)_I U^I{}_5 \Psi_R + a_1 y_R f (\bar{t}_R^5)_I U^I{}_5 \Psi_L + h.c. \right]$$

The non-linear realization of $SO(5)$ ($g \in SO(5), h(x) \in SO(4)$):

$$U^I{}_i \to g^I{}_J h_i^{*j} U^J{}_j, \qquad \boxed{U^I{}_5 \to g^I{}_J U^J{}_5}$$

The constrained $SO(5)$ vector:

$$\Sigma^I = U^I{}_5 = (0,0,0,s_h,c_h)^T \,, \qquad \boxed{\Sigma^\dagger \Sigma = 1}$$

$$\boxed{SO(4) \simeq SU(2)_L \times SU(2)_R, \quad Y = T^{3R} + X}$$

The embedding of the SM third-generation quark:

$$q_L^5 = t_L P_{t_L} + b_L P_{b_L}, \quad \bar{q}_L^5 = \bar{t}_L P_{t_L}^\dagger + \bar{b}_L P_{b_L}^\dagger, \quad t_R^5 = t_R P_{t_R}, \quad \bar{t}_R^5 = \bar{t}_R P_{t_R}^\dagger$$

The vectors are determined by their SM quantum numbers:

$$(P_{t_L})^I = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ i \\ -1 \\ 0 \end{pmatrix}, \qquad (P_{b_L})^I = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad (P_{t_R})^I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Upper indices transform under g, lower indices under g^*

Partial compositeness

The decomposition of the fourplet:

$$\Psi_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} iB - iX_{5/3} \\ B + X_{5/3} \\ iT + iX_{2/3} \\ -T + X_{2/3} \end{pmatrix} \qquad \Psi_1 = \tilde{T}.$$

Two $SU(2)_L$ doublets:

$$q_T = (T, B)_{1/6}, \qquad q_X = (X_{5/3}, X_{2/3})_{7/6}$$

Before EWSB:

$$c_4 y_L f \bar{q}_L q_{TR}, \qquad a_1 y_R f \bar{t}_R \tilde{T}_L$$

The mixing angles:

$$\tan\theta_L = \frac{c_4 y_L f}{M_4}, \qquad \tan\theta_R = \frac{a_1 y_R f}{M_1}$$

Spurion Analysis

$$\boxed{\mathcal{G} = SO(5) \times U(1)_X \times U(1)^3_{el}}$$

The embedding vectors are treated as spurions:

$$(P_q)^I \to g^I{}_J (P_q)^J, \qquad (P_q^\dagger)_I \to g^{*J}_I (P_q^\dagger)_J$$

The elementary $U(1)^3_{el}$ global symmetry $q = (t_L, b_L, t_R)$:

$$q \to e^{i\alpha_q} q, \quad P_q \to e^{-i\alpha_q} P_q$$

$$\mathcal{L}^{M4_5} = -M_4 \bar{\Psi}\Psi + \left[c_4 y_L f (\bar{q}^5_L)_I U^I{}_i \Psi^i_R + a_4 y_R f (\bar{t}^5_R)_I U^I{}_i \Psi^i_L + h.c. \right]$$

$$\mathcal{L}^{M1_5} = -M_1 \bar{\Psi}\Psi + \left[c_1 y_L f (\bar{q}^5_L)_I U^I{}_5 \Psi_R + a_1 y_R f (\bar{t}^5_R)_I U^I{}_5 \Psi_L + h.c. \right]$$

Spurion Analysis

$$\boxed{\mathcal{G} = SO(5) \times U(1)_X \times U(1)^3_{el}}$$

$$\Pi_{t_L}\ \bar{t}_L \not{p}\ t_L + \Pi_{b_L}\ \bar{b}_L \not{p}\ b_L + \Pi_{t_R}\ \bar{t}_R \not{p}\ t_R - (\Pi_{t_L t_R}\ \bar{t}_L\ t_R + \mathrm{h.c.})$$

The form factors are determined by the invariants:

$$P^\dagger_{t_L}\Sigma\Sigma^\dagger P_{t_L} = \frac{s_h^2}{2}, \quad \boxed{P^\dagger_{b_L}\Sigma\Sigma^\dagger P_{b_L} = 0}, \quad P^\dagger_{t_R}\Sigma\Sigma^\dagger P_{t_R} = c_h^2$$

$$\boxed{P^\dagger_{t_L}\Sigma\Sigma^\dagger P_{t_R} = -\frac{s_h c_h}{\sqrt{2}}}$$

$$\Downarrow$$

$$\Pi_{t_L} = \Pi_{0t_L} + s_h^2\ \Pi_{1t_L}, \qquad \Pi_{t_R} = \Pi_{0t_R} + s_h^2\ \Pi_{1t_R}$$

$$\Pi_{t_L t_R} = s_h c_h \Pi_{1 t_L t_R}$$

The couplings

$$c_{t,g} = 1 + \Delta_{t,g}\xi + \cdots, \quad \xi = v^2/f^2, \quad r_1 = \frac{c_4 a_4}{c_1 a_1}\frac{M_1}{M_4}$$

The *ggh* coupling strength:

$$\Pi_{t_L t_R}(0) \propto s_h c_h \Rightarrow c_g^{(t)} = \sin\theta \frac{\partial}{\partial\theta} \log \Pi_{t_L t_R}(0) = \frac{\cos 2\theta}{\cos\theta}$$

$$\Downarrow$$

$$\Delta_g = -\tfrac{3}{2}$$

The top Yukawa coupling:

$$\Delta_t - \Delta_g = \frac{1}{2}\left(1 - \frac{1}{r_1^2}\right)\sin^2\theta_L + \left(1 - r_1^2\right)\sin^2\theta_R < 1$$

$$\Downarrow$$

$$\Delta_t < -1/2$$

14 of $SO(5)/SO(4)$

$$\boxed{\mathbf{14} = \mathbf{9}(3,3) \oplus \mathbf{4}(2,2) \oplus \mathbf{1}}$$

The effective Lagrangian:

$$\mathcal{L}^{M9_{14}} = -M_9 \bar{\Psi}_{ij}\Psi^{ij} + \left[c_9 y_L f (\bar{q}_L^{14})_{IJ} U^I{}_i U^J{}_j \Psi_R^{ij} + a_9 y_R f (\bar{t}_R^{14})_{IJ} U^I{}_i U^J{}_j \Psi_L^{ij} + h.c. \right]$$

$$\mathcal{L}^{M4_{14}} = -M_4 \bar{\Psi}\Psi + \sqrt{2} \left[c_4 y_L f (\bar{q}_L^{14})_{IJ} U^I{}_i U^J{}_5 \Psi_R^i + a_4 y_R f (\bar{t}_R^{14})_{IJ} U^I{}_i U^I{}_5 \Psi_L^i + h.c. \right]$$

$$\mathcal{L}^{M1_{14}} = -M_1 \bar{\Psi}\Psi + \frac{\sqrt{5}}{2} \left[c_1 y_L f (\bar{q}_L^{5})_{IJ} U^I{}_5 U^J{}_5 \Psi_R + a_1 y_R f (\bar{t}_R^{14})_{IJ} U^I{}_5 U^J{}_5 \Psi_L + h.c. \right]$$

The SM quark embedding matrices:

$$(P_{t_L})^{IJ} = \frac{1}{2} \begin{pmatrix} & & & & 0 \\ & & & & 0 \\ & & & & i \\ & & & & -1 \\ 0 & 0 & i & -1 & \end{pmatrix}, \quad (P_{b_L})^{IJ} = \frac{1}{2} \begin{pmatrix} & & & & i \\ & & & & 1 \\ & & & & 0 \\ & & & & 0 \\ i & 1 & 0 & 0 & \end{pmatrix},$$

$$(P_{t_R})^{IJ} = \frac{1}{2\sqrt{5}} \mathrm{diag}(-1,-1,-1,-1,4)$$

The invariants

The advantage of **14** is that now we have two types of invariants:

$$\boxed{\Sigma^T P_q^\dagger P_q \Sigma^*, \qquad \Sigma^T P_q^\dagger \Sigma\ \Sigma^\dagger P_q \Sigma^*}$$

The invariants affecting the *ggh* coupling:

$$\Sigma^T P_{t_L}^\dagger P_{t_R} \Sigma^* = -\frac{3}{4\sqrt{5}} s_h c_h,$$

$$\Sigma^T P_{t_L}^\dagger \Sigma\ \Sigma^\dagger P_{t_R} \Sigma^* = -\frac{2\sqrt{5}}{5} s_h c_h + \frac{\sqrt{5}}{2} s_h^3 c_h$$

$$\Downarrow$$

$$\Pi_{t_L t_R} = s_h c_h \left(\Pi_{1 t_L t_R} + s_h^2\ \Pi_{2 t_L t_R} + \cdots\right)$$

$$\Downarrow$$

$$\Delta_g^{(t)} = -\frac{3}{2} + \frac{2\ \Pi_{2 t_L t_R}}{\Pi_{1 t_L t_R}}$$

The Higgs couplings

$$r_1 = \frac{c_4 a_4}{c_1 a_1}\frac{M_1}{M_4}, \quad r_9 = \frac{c_4 a_4}{c_9 a_9}\frac{M_9}{M_4}, \quad r_1^2 = \frac{M_1^2}{M_4^2}, \quad r_9^2 = \frac{M_9^2}{M_4^2}$$

The *ggh* coupling depends on the mass scales now:

$$\Delta_g^{(t)} = -\frac{3}{2}\frac{1-1/r_9}{1-1/r_1} - 4, \qquad \Delta_g^{(b)} = \left(\frac{1}{r_9^2} - 1\right)\sin^2\theta_L$$

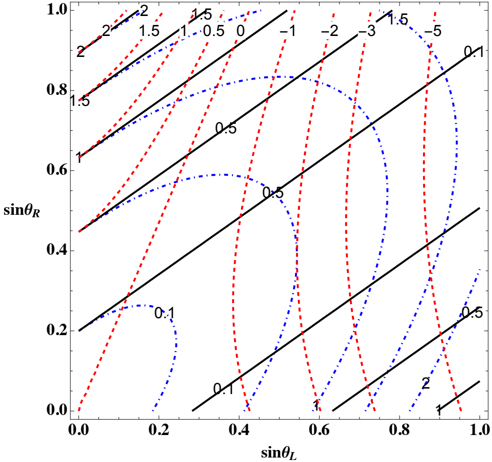
The modification to the top Yukawa:

$$\Delta_t = \Delta_g^{(t)} + \frac{5}{2}\sin^2\theta_L\left(1 - \frac{1}{2r_9^2} - \frac{1}{2r_1^2}\right) + \frac{5}{2}\sin^2\theta_R\left(1 - r_1^2\right)$$

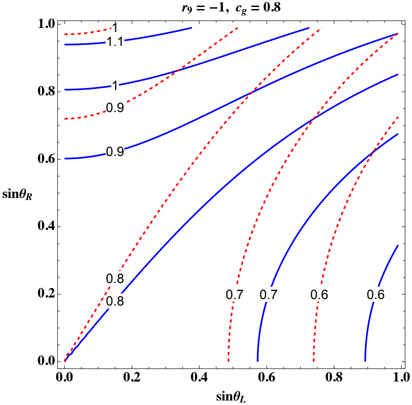
The difference $\Delta_t - \Delta_g$

The essential quantity to fit the data:

$$\Delta_t - \Delta_g = \frac{1}{4}\sin^2\theta_L\left(14 - \frac{9}{r_9^2} - \frac{5}{r_1^2}\right) + \frac{5}{2}\sin^2\theta_R\left(1 - r_1^2\right)$$



Benchmark plot: $r_9 = -1$



Red dashed lines: $\xi = 0.1, r_1 \sim 0.4$

Blue solid lines: $\xi = 0.2, r_1 \sim 0.5$

Higgs potential

The Higgs potential:

$$V_f(h) \simeq \frac{N_c M_4^4}{16\pi^2}\left(-\tilde{\gamma}_f \sin^2\frac{h}{f} + \tilde{\beta}_f \sin^4\frac{h}{f}\right)$$

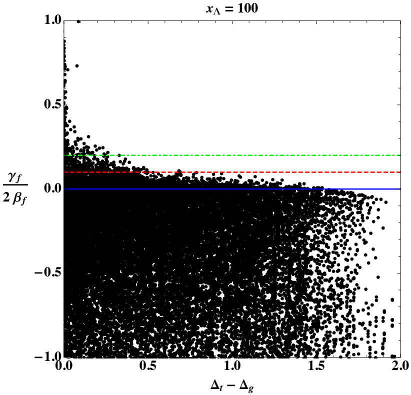
We can see the tension between positive $\Delta_t - \Delta_g$ and positive γ_f:

$$\begin{aligned}
\Delta_t - \Delta_g &= \frac{9}{4}\sin^2\theta_L\left(1-\frac{1}{r_9^2}\right) + \frac{5}{4}\sin^2\theta_L\left(1-\frac{1}{r_1^2}\right) + \frac{5}{2}\sin^2\theta_R(1-r_1^2) \\
\tilde{\gamma}_f^{y_L^2} &= \frac{9}{2}\tan^2\theta_L(1-r_9^2)\left[f_1(x_\Lambda,\sec^2\theta_L) - r_9^2 f_2(x_\Lambda, r_9^2, \sec^2\theta_L)\right] \\
&\quad + \frac{5}{2}\tan^2\theta_L(1-r_1^2)\left[f_1(x_\Lambda,\sec^2\theta_L) - r_1^2 f_2(x_\Lambda, r_1^2, \sec^2\theta_L)\right] \\
\tilde{\gamma}_f^{y_R^2} &= -5\tan^2\theta_R\ r_1^2(1-r_1^2)\left[f_1(x_\Lambda, r_1^2\sec^2\theta_R) - f_2(x_\Lambda, 1, r_1^2\sec^2\theta_R)\right]
\end{aligned}$$

$$\boxed{x_\Lambda = \Lambda^2/M_4^2}$$

Including $\gamma_f^{y^4}$

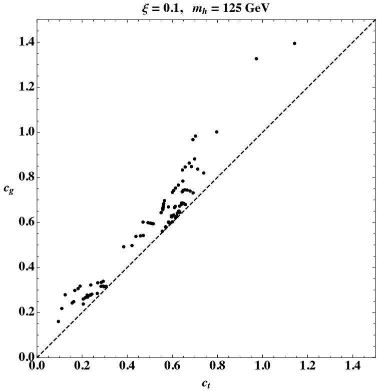
$$r_{1,9} \in [-2, 2], \quad \theta_{L,R} \in [0, \pi/2]$$



The Higgs mass

After EWSB:

$$\xi = \frac{\tilde{\gamma}_f}{2\tilde{\beta}_f}, \quad m_h^2 = \frac{8N_c M_4^4}{16\pi^2 f^2}\tilde{\beta}_f \xi(1-\xi) \sim (158\,\mathrm{GeV})^2 \left(\frac{M_4}{1\,\mathrm{TeV}}\right)^2 \left(\frac{\xi}{0.1}\right)^2 \tilde{\beta}_f$$



$$m_t = 150\,\mathrm{GeV}, \qquad \Lambda = 4\pi f \sim 9.8\,\mathrm{TeV}$$

Possible solution

Enlarge to $SO(6)/SO(5)$, gauge an extra $U(1)_A$ in the coset:

$$T^{\hat{5}}_{IJ} = -\frac{i}{\sqrt{2}}(\delta^{5I}\delta^{6J} - \delta^{5J}\delta^{6I})$$

We have the contribution from the gauge sector:

$$\gamma_g = \frac{c\, m_\rho^4}{64\pi^2}\left(2\frac{g_A^2}{g_\rho^2} - 3\frac{g^2}{g_\rho^2} - \frac{g'^2}{g_\rho^2}\right)$$

Need careful study!

See also:

M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B **254** (1985) 299.

Conclusion

- LHC data prefer $c_t > c_g$.
- We find strong correlation between $c_t - c_g$ and the Higgs mass term in the composite Higgs framework.
- Possible $c_t - c_g$ usually leads to positive Higgs mass term without EWSB.
- An extra $U(1)_A$ gauge boson may solve the problem.

The form factor (continue)

$$\Pi_{2t_L}(p^2) = \frac{1}{4} y_L^2 f^2 \left(3\frac{c_9^2}{p^2 - M_9^2} - 8\frac{c_4^2}{p^2 - M_4^2} + 5\frac{c_1^2}{p^2 - M_1^2} \right)$$

$$\Pi_{2t_R}(p^2) = \frac{5}{16} y_R^2 f^2 \left(-3\frac{a_9^2}{p^2 - M_9^2} + 8\frac{a_4^2}{p^2 - M_4^2} - 5\frac{a_1^2}{p^2 - M_1^2} \right)$$

$$\Pi_{1t_L t_R}(p^2) = \frac{\sqrt{5}}{2} y_L y_R f^2 \left(\frac{c_4 a_4 M_4}{p^2 - M_4^2} - \frac{c_1 a_1 M_1}{p^2 - M_1^2} \right)$$

$$\Pi_{2t_L t_R}(p^2) = \frac{\sqrt{5}}{8} y_L y_R f^2 \left(3\frac{c_9 a_9 M_9}{p^2 - M_9^2} - 8\frac{c_4 a_4 M_4}{p^2 - M_4^2} + 5\frac{c_1 a_1 M_1}{p^2 - M_1^2} \right)$$