Deciphering the Majorana nature of sub-eV neutrinos by using their quantum statistical property

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Contents

Introduction

Neutrinos have mass.

- In the SM, neutrinos are
  - massless ($m_\nu = 0$) and
  - only left-handed (ν_L). No right-handed neutrinos (ν_R) observed yet.
- Giving neutrinos mass in the SM is possible iff we introduce ν_R,

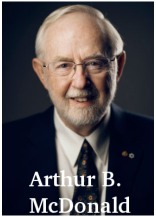
$$\mathscr{L}_{\mathrm{SM}} \supset -m_\nu \left(\overline{\nu_R}\, \nu_L + \overline{\nu_L}\, \nu_R \right), \quad \text{where} \quad m_\nu = \frac{Y_\nu \, \mathrm{v}}{\sqrt{2}},$$

$$Y_\nu = \text{Higgs-neutrino Yukawa coupling constant,}$$

$$\mathrm{v} = \text{Higgs VEV.}$$

- neutrino oscillation $\implies m_\nu \neq 0$.
- **Nobel Prize in Physics 2015**

"for the discovery of neutrino oscillations, which shows that neutrinos have mass".



There are various suggestions as to how neutrinos can get mass.

- **Dirac mass:**
  - *Assumption:* ν_R exists.
  - *Lagrangian:*
  $$\mathscr{L}^{D}_{\text{mass}} = -m^{D}_{\nu} \left(\overline{\nu_R}\, \nu_L + \overline{\nu_L}\, \nu_R \right).$$
  - *Disadvantage:* No reason for m^D_ν to be small.
  - *Challenge:* Finding ν_R.

- **Majorana mass:**
  - *Assumption:* neutrino $\equiv$ anti-neutrino.
  - *Lagrangian:*
  $$\mathscr{L}^{M}_{\text{mass}} = \tfrac{1}{2} m^{M}_{\nu} \left(\overline{\nu^{C}_{L}}\, \nu_L + \overline{\nu_L}\, \nu^{C}_{L} \right).$$
  - *Disadvantage:* $\mathscr{L}^{M}_{\text{mass}}$ is not invariant under $SU(2)_L \times U(1)_Y$ gauge group $\therefore \mathscr{L}^{M}_{\text{mass}}$ is not allowed by SM.
  - *Challenge:* To ascertain the Majorana nature of light neutrino.

Introduction

How to give neutrinos mass?

- **Dirac-Majorana mass:**
  - *Assumptions:* ν_R exists, and neutrino $\equiv$ anti-neutrino.
  - *Lagrangian:*

$$\mathscr{L}_{\text{mass}}^{D+M} = \frac{1}{2} m_\nu^L \left(\overline{\nu_L^C} \nu_L \right) + \frac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R \right) - m_\nu^D \left(\overline{\nu_R} \nu_L \right) + \text{H.c.}$$
$$= \frac{1}{2} \left(m_1 \overline{\nu_1^C} \nu_1 + m_2 \overline{\nu_2^C} \nu_2 \right) + \text{H.c.},$$

where $\nu_k = \nu_{kL} + \nu_{kL}^C$ $(k = 1, 2)$ are Majorana neutrinos and

$$m_{2,1} = \frac{1}{2} \left(\left(m_\nu^L + m_\nu^R \right) \pm \sqrt{\left(m_\nu^L + m_\nu^R \right)^2 + 4 \left| m_\nu^D \right|^2} \right).$$

  - *Disadvantages:*
    - No explanation for small mass of neutrinos, and
    - one mass out of m_ν^D, m_ν^R, m_ν^L is complex, in general.
  - *Challenges:*
    - To prove that both ν_1 and ν_2 are Majorana neutrinos.
    - What is the physical meaning of complex mass?

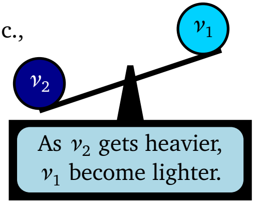
Introduction

How to give neutrinos mass?

- **See-saw mechanism:** A simpler version of Dirac-Majorana mass, with a nice twist.
  - *Assumptions:* $m_\nu^L = 0$ and $m_\nu^D \ll m_\nu^R$.
  - *Lagrangian:*

$$\begin{aligned}\mathscr{L}_{\text{mass}}^{D+M} &= \tfrac{1}{2} m_\nu^R \left(\overline{\nu_R^C}\,\nu_R\right) - m_\nu^D \left(\overline{\nu_R}\,\nu_L\right) + \text{H.c.} \\ &= \tfrac{1}{2}\left(m_1 \overline{\nu_1^C}\,\nu_1 + m_2 \overline{\nu_2^C}\,\nu_2\right) + \text{H.c.},\end{aligned}$$

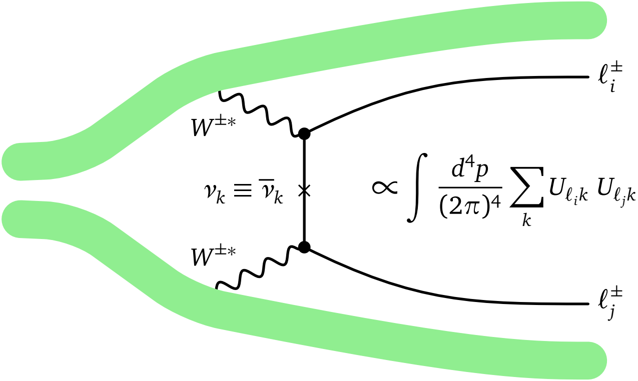
where $m_1 \approx -\dfrac{\left(m_\nu^D\right)^2}{m_\nu^R}$ and $m_2 \approx m_\nu^R$.



  - *Advantage:* $m_1 \ll m_2 \implies \nu_1$ is a light neutrino
  - *Challenges:*
    - To find the heavy ν_2 experimentally.
    - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

- Neutrinos: the only known *elementary fermions* that *can* have Majorana nature ($\nu \equiv \overline{\nu}$).
- Majorana neutrino: very unique phenomenology (lepton number non-conservation), they mediate $\Delta L = 2$ processes.



$$\propto \int \frac{d^4p}{(2\pi)^4} \sum_k U_{\ell_i k}\, U_{\ell_j k}\, \frac{m_k + \not{p}}{p^2 - m_k^2}$$

- $\Delta L = 2$ processes play crucial role to probe Majorana nature of ν's.
  - neutrinoless double-beta ($0\nu\beta\beta$) decay
  - Rare meson decays with $\Delta L = 2$
  - Collider searches at LHC

Looking for Majorana neutrinos via $\Delta L = 2$ processes

- **Decay rate of any $\Delta L = 2$ process with final leptons $\ell_1^+ \ell_2^+$:**

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} \frac{m_k}{p^2 - m_k^2 + i m_k \Gamma_k} \right|^2 ,$$

where we have used the fact that $(1-\gamma^5)\,\not{p}(1-\gamma^5) = 0$.

- Light ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} m_k \right|^2 = \left| m_{\ell_1 \ell_2} \right|^2 .$$

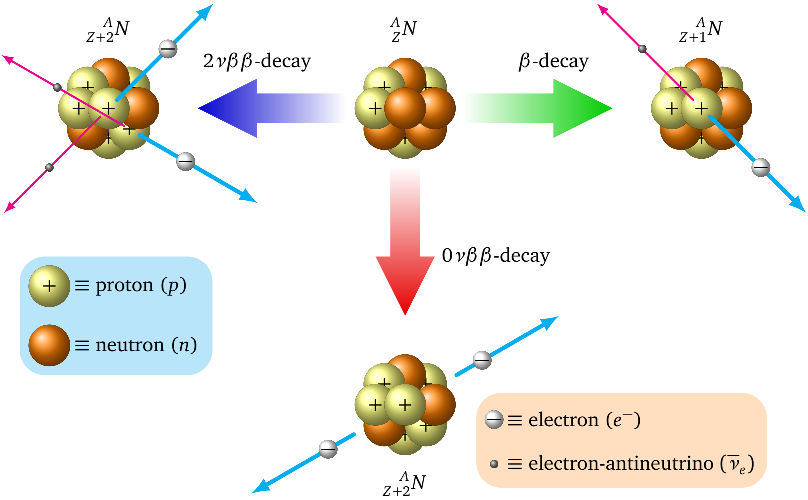
- Heavy ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k \frac{U_{\ell_1 k} U_{\ell_2 k}}{m_k} \right|^2 .$$

- Resonant ν:

$$\Gamma_{\Delta L=2} \propto \frac{\Gamma(N \to i)\,\Gamma(N \to f)}{m_N \Gamma_N} .$$

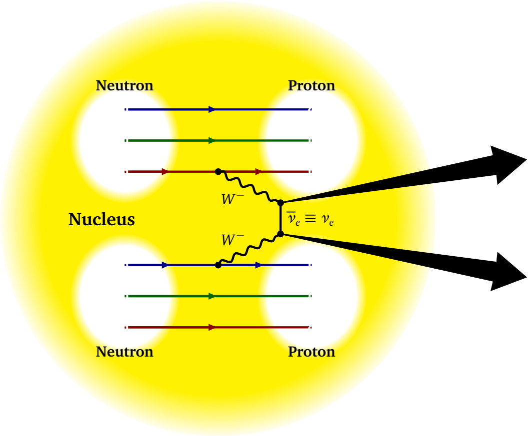
A comparison with beta decay and double-beta decay:



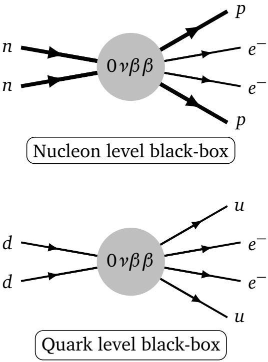
Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay

Mechanism for $0\nu\beta\beta$ decay:



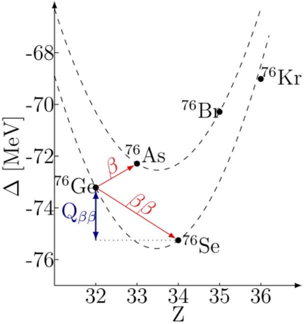
Black-box diagrams for $0\nu\beta\beta$:

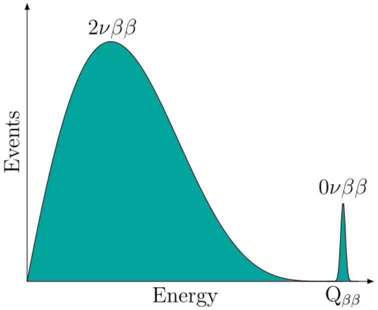


Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay

- Double-beta ($2\nu\beta\beta$) decay has been observed in 10 isotopes, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, with half-life $T_{1/2} \approx 10^{18} - 10^{24}$ years.





Giovanni Benato (for the GERDA collaboration), arXiv:1509.07792

- **$0\nu\beta\beta$ (forbidden in SM) is yet to be observed in any experiment.**

$$T_{1/2}^{0\nu}\left[{}^{76}\mathrm{Ge}\right] > 2.1 \times 10^{25} \text{ years} \,(90\% \text{ C.L.})\,.$$

M. Agostini *et al.* (GERDA Collaboration) Phys. Rev. Lett. **111**, 122503 (2013).

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay

❖ The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

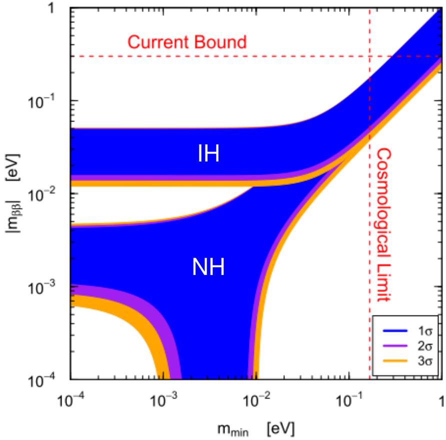
$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\ |M_{0\nu}|\ \left|m_{\beta\beta}\right|^2,$$

where

- $G_{0\nu}$ is phase space factor,
- $M_{0\nu}$ is the *nuclear matrix element*, (large theoretical uncertainty)
- $m_{\beta\beta}$ is *effective Majorana mass*. $m_{\beta\beta} = \sum_{k=1}^{3} U_{ek}^2 m_k$ is complex, in general, and can be zero due to possible cancellations arising from phases in U_{ek}.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay



NH: Normal hierarchy
IH: Inverted hierarchy

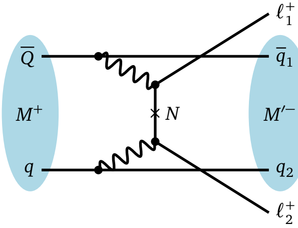
S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012), arXiv:1203.5250

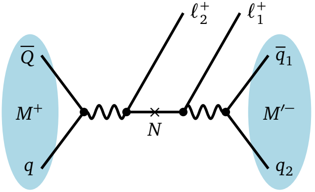
❖ **If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T^{0\nu}_{1/2}$ will be much larger than the current experimental lower bound.**

- **Processes:** $M^+ \to M'^- \ell_1^+ \ell_2^+$,
  where $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, \ldots$

G. Cvetic, C.S. Kim, arXiv:1606.04140 (PRD **94**, 053001, 2016)

G. Cvetic, C. Dib, S. Kang, C. S. Kim, arXiv:1005.4282 (PRD **82**, 053010, 2010)

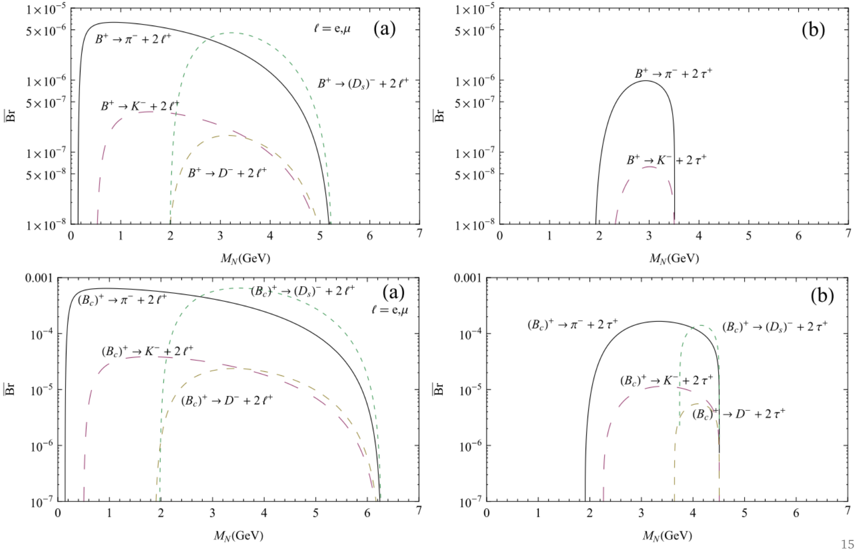




- **No nuclear matrix element unlike** $0\nu\beta\beta$, but probes Majorana nature of massive neutrino(s) N.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Rare meson decays: $M^+ \to M'^- \ell_1^+ \ell_2^+$



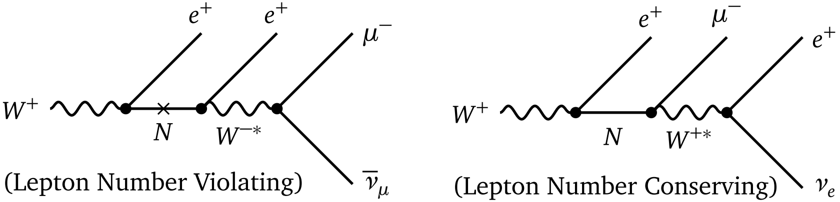
Looking for Majorana neutrinos via $\Delta L = 2$ processes

Collider searches at LHC

- **Processes:** $W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu$, $W^+ \to \mu^+ \mu^+ e^- \overline{\nu}_e$. Involves heavy neutrino N which can have Majorana nature as well.

C. Dib, C.S. Kim, arXiv:1509.05981 (PRD **92**, 093009, 2015);

C. Dib, C.S. Kim, K. Wang, J. Zhang, arXiv:1605.01123 (PRD **94**, 013005, 2016)



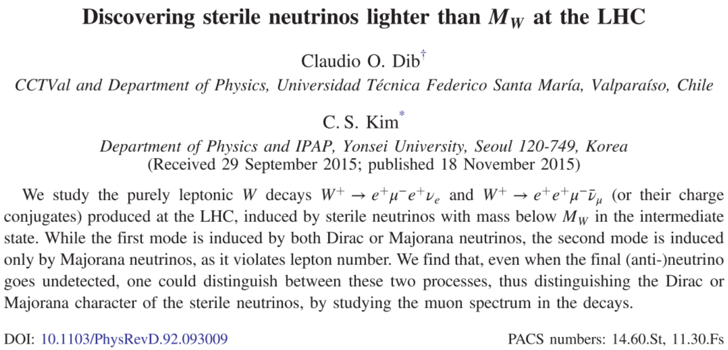
(Lepton Number Violating) (Lepton Number Conserving)

- **Decay widths:**
  - LNV: $\Gamma\left(W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu\right) = |U_{Ne}|^4\, \hat{\Gamma}$,
  - LNC: $\Gamma\left(W^+ \to e^+ e^+ \mu^- \overline{\nu}_\mu\right) = \left|U_{Ne} U_{N\mu}\right|^2 \hat{\Gamma}$,

where $\hat{\Gamma} = \dfrac{G_F^3 M_W^3}{12 \times 96\sqrt{2}\pi^4} \dfrac{m_N^5}{\Gamma_N} \left(1 - \dfrac{m_N^2}{M_W^2}\right)^2 \left(1 - \dfrac{m_N^2}{2M_W^2}\right)$.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Collider searches at LHC



Discovering sterile neutrinos lighter than M_W at the LHC

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(Received 29 September 2015; published 18 November 2015)

We study the purely leptonic W decays $W^+ \to e^+\mu^- e^+\nu_e$ and $W^+ \to e^+e^+\mu^-\bar{\nu}_\mu$ (or their charge conjugates) produced at the LHC, induced by sterile neutrinos with mass below M_W in the intermediate state. While the first mode is induced by both Dirac or Majorana neutrinos, the second mode is induced only by Majorana neutrinos, as it violates lepton number. We find that, even when the final (anti-)neutrino goes undetected, one could distinguish between these two processes, thus distinguishing the Dirac or Majorana character of the sterile neutrinos, by studying the muon spectrum in the decays.

DOI: 10.1103/PhysRevD.92.093009 PACS numbers: 14.60.St, 11.30.Fs

APS News (Nov 18, 2015) for
"Physics – Spotlighting Exceptional Research"

http://physics.aps.org/synopsis-for/10.1103/PhysRevD.92.093009

Particles & Fields
SYNOPSIS: LHC Data Might Reveal Nature of Neutrinos

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Collider searches at LHC

Synopsis: LHC Data Might Reveal Nature of Neutrinos

November 18, 2015

A long-standing question over whether the neutrino is its own antiparticle might be answered by looking at decays of *W* bosons.

As recognized by this year's Nobel Prize in physics, evidence now points to neutrinos having mass (see **7 October 2015 Focus** story). But this opens up new questions about why the neutrino mass is so much smaller than other particle masses. One solution is to assume that the neutrino is a different kind of particle—one that is its own antiparticle. A new theoretical study shows that observations of *W* boson decays at the Large Hadron Collider (LHC) in Geneva could potentially uncover the antiparticle nature of the neutrino.

Electrons, protons, and other fermions are Dirac particles, meaning they have a separate antiparticle with the same mass, but opposite charge. Neutrinos could be Dirac particles, but because they have no electric charge, they could also be Majorana particles, for which particle and antiparticle are the same thing. Such Majorana models are attractive because they offer a fairly natural explanation for the extremely small neutrino mass.

Experiments looking at extremely rare nuclear decays are trying to detect a possible Majorana or Dirac signature of the neutrino. To widen the search, Claudio Dib from Santa María University in Chile and Choong Sun Kim from Yonsei University in Korea propose looking at *W* boson decays. They considered decays that result in specific combinations of electrons, muons, and neutrinos. These decays have yet to be observed, but they are predicted in theories involving hypothetical sterile neutrinos. Taking into account current limits on the existence of sterile neutrinos, the team predicts that the next runs at the LHC could produce as many as a few thousand of the desired *W* boson decays. If this count is correct, then physicists should be able to discriminate Majorana from Dirac neutrinos by the shape of the energy spectrum of the outgoing muons.

This research is published in **Physical Review D.**

–Michael Schirber

The 'practical Dirac-Majorana confusion theorem'

- **Practical Dirac-Majorana confusion theorem:** By looking at the total decay rate or any other kinematic test of a process allowed in the SM, it is practically impossible to distinguish between the Dirac and Majorana neutrinos in the limit neutrino mass goes to zero.

  B. Kayser, Phys. Rev. D **26**, 1662 (1982).

- **Conceptual basis:** Let $m_\nu =$ mass of neutrino (ν).
  - When $m_\nu \neq 0$, ν is *not* a chirality eigenstate, but a helicity eigenstate.
  - In terms of helicity,
    - a Dirac neutrino ν^D has four states ν^D_-, $\overline{\nu}^D_+$, ν^D_+ and $\overline{\nu}^D_-$, but
    - a Majorana neutrino ν^M always has two states ν^M_+ and ν^M_-.
  - When $m_\nu \to 0$, its difficult to distinguish between $\left(\nu^D_-, \overline{\nu}^D_+\right) \approx (\nu_L, \overline{\nu}_R)$ and $\left(\nu^M_-, \nu^M_+\right) \approx (\nu_L, \overline{\nu}_R)$.
  - When $m_\nu \neq 0$, ν_L can behave like a $\nu_R \implies$ all differences between ν^D and ν^M present in a kinematic test *suppressed* by $(m_\nu/E_\nu)^x$, where E_ν is the energy of neutrino, and x is some power.

The ‘practical Dirac-Majorana confusion theorem’

How to overcome this hurdle in deciphering the Majorana nature of neutrinos?

Majorana condition:
$\nu_\ell \equiv \overline{\nu}_\ell,\ \ell \in \{e, \mu, \tau\}$

Independent of m_ν

Fermi-Dirac statistics:
quantum mechanical identicalness of ν_ℓ and $\overline{\nu}_\ell$

Processes with final state containing $\nu_\ell, \overline{\nu}_\ell$ in addition to other particles

amplitude must be *antisymmetrized* under $\nu_\ell \longleftrightarrow \overline{\nu}_\ell$ exchange

To *probe* this exchange symmetry, we need to know the 4-momenta of ν_ℓ and $\overline{\nu}_\ell$

Need to *find a way* to infer the 4-momenta of ν_ℓ and $\overline{\nu}_\ell$

⚠ current experimental facilities might not be fully optimised for such study

Any kinematical observable which includes integration over 4-momenta of $\nu_\ell, \overline{\nu}_\ell$ can’t be used for this study

Kinematic observables under **practical Dirac-Majorana confusion theorem** include integration over 4-momenta of ν_ℓ and $\overline{\nu}_\ell$.
Difference between Dirac and Majorana cases in such observables are suppressed by some power of (m_ν/E_ν).

Statistical Nature of Neutrinos

So far only the interaction properties of Majorana neutrinos have been exploited.

- Majorana neutrinos are a theorists favourite, because of their simplicity and the resulting elegance in theory, with exception of the nuclear matrix element in $0\nu\beta\beta$.

- All major searches for Majorana neutrinos, for both active and heavy neutrino cases, have exploited only their **mass dependent interaction property**, $\mathscr{L}_{\text{int}} = m_\nu \overline{\nu}\nu$.
  $\therefore m_\nu = 0 \implies$ no $0\nu\beta\beta$ decay or other $\Delta L = 2$ processes.

- We want a better alternative to $0\nu\beta\beta$ decay or other $\Delta L = 2$ processes. These alternative processes,
  - should not be rare, and
  - must have a unique, experimentally observable signature for Majorana neutrinos.

- We shall explore the quantum statistical property of Majorana neutrinos which is independent of neutrino mass.

Statistical Nature of Neutrinos

The quantum statistical property of Majorana neutrinos does not depend upon their masses.

- $\because$ quantum statistical property does not depend on mass, any kinematic test designed to *directly probe* the exchange symmetry of Majorana neutrino and anti-neutrino *must not* be dependent on mass of neutrino.

- To directly probe the $\nu_\ell \longleftrightarrow \overline{\nu}_\ell$ exchange symmetry for Majorana case, their 4-momenta must be deduced experimentally. This might be possible if we could *directly* measure the 4-momentum of some intermediate resonances.

- Our choice of decay modes and our kinematic observables are guided keeping the following requirements in mind,
  - 4-momenta of ν_ℓ and $\overline{\nu}_\ell$ must be deducible,
  - observable must explicitly check $\nu_\ell \longleftrightarrow \overline{\nu}_\ell$ exchange symmetry,
  - the difference between Dirac and Majorana cases should be very distinct.

'Effective' Daitz plot method

We consider only such decay modes in which the 4-momentum of neutrino and anti-neutrino can be experimentally inferred.

- **Example decay mode:** $B^0 \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu$
  - 4-momentum of B^0 is routinely measured by looking at 4-momentum of the fully tagged $\bar{B}^0$ arising in the process $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$.
  - 4-momentum of μ^+ and μ^- are also routinely observed experimentally.
  - Assuming that 4-momentum of charged pion can be deduced independent of the subsequent muon decay, the 4-momenta of ν_μ and $\overline{\nu}_\mu$ can be known by applying conservation of 4-momentum.
  - We analyse this 4-body final state as an 'effective' 3-body final state by treating $\mu^+\mu^-$ as an 'effective' third particle.
  - We work in a frame of reference in which the exchange between ν_ℓ and $\overline{\nu}_\ell$ is very easy to visualise.

'Effective' Daitz plot method

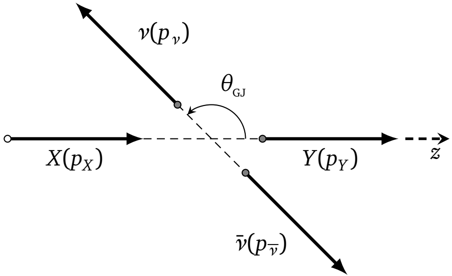
We consider only such decay modes in which the 4-momentum of neutrino and anti-neutrino can be experimentally inferred.

- **Some more example decay modes:**
  - $X\left[B^0\right] \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$,
  - $X\left[B_s^0\right] \to K^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$,
  - $X\left[B^0\right] \to \pi^+\left(\to \mu^+\nu_\mu\right)\pi^-\left(\to \mu^-\overline{\nu}_\mu\right) \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.
- **Process:** $X \to Y\left[\ell^+\ell^-\mathscr{Y}\right]\nu_\ell\overline{\nu}_\ell$ (an 'effective' three-body decay)
- **Conditions:**
  1. X is some suitable resonance.
  2. Y is an 'effective' particle, which must always include $\ell^+\ell^-$, with some additional (not necessary) particle(s) $\mathscr{Y}$.
  3. The 4-momenta of X and all particles in Y as well as those of some *intermediate resonances* must be experimentally measured such that 4-momenta of ν_ℓ and $\overline{\nu}_\ell$ are experimentally deducible.
- A tentative list of many more decay modes that can be used in our study, will be shown before the numerical results.

‘Effective’ Daitz plot method

We choose to work in a frame of reference in which exchange of ν and $\overline{\nu}$ is more elegant.

Gottfried-Jackson frame: $\vec{p}_\nu + \vec{p}_{\overline{\nu}} = \vec{0}$.



Invariant mass squares:

$$m^2_{\nu\overline{\nu}} = (p_\nu + p_{\overline{\nu}})^2 = (p_X - p_Y)^2,$$
$$m^2_{Y\nu} = (p_Y + p_\nu)^2 \equiv a - b\cos\theta_{\mathrm{GJ}},$$
$$m_{Y\overline{\nu}} = (p_Y + p_{\overline{\nu}})^2 \equiv a + b\cos\theta_{\mathrm{GJ}}.$$

where

$$a = \frac{1}{2}\left(m_X^2 + m_Y^2 + 2m_\nu^2 - s\right),$$
$$b = \frac{1}{2}\left(\sqrt{\lambda\left(m_X^2, m_Y^2, s\right)\left(1 - 4m_\nu^2/s\right)}\right),$$

with the Källén function $\lambda(x,y,z)$ defined as

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx).$$

‘Effective’ Daitz plot method

Our tool for investigating the Majorana neutrinos is the ‘effective’ Dalitz plot.

- $m_{\nu\overline{\nu}}^2 + m_{Y\nu}^2 + m_{Y\overline{\nu}}^2 = m_X^2 + m_Y^2 + 2m_\nu^2 \equiv M^2$ (say). Since m_Y^2 varies from event-to-event, M^2 does so also.

- Define new dimensionless ratios to take care of these event-to-event variations, $\tilde{m}_{\nu\overline{\nu}}^2 \equiv m_{\nu\overline{\nu}}^2/M^2$, $\tilde{m}_{Y\nu}^2 \equiv m_{Y\nu}^2/M^2$, $\tilde{m}_{Y\overline{\nu}}^2 \equiv m_{Y\overline{\nu}}^2/M^2$, such that $\tilde{m}_{\nu\overline{\nu}}^2 + \tilde{m}_{Y\nu}^2 + \tilde{m}_{Y\overline{\nu}}^2 = 1$.

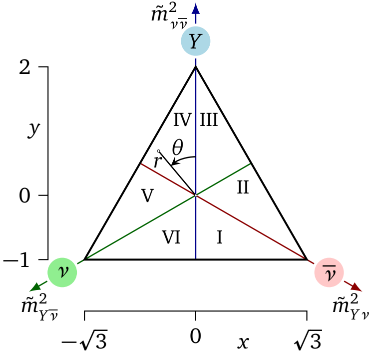
- We can always construct a ternary plot (which along with event points we shall refer to as the ‘effective’ Dalitz plot) using $\left(\tilde{m}_{Y\nu}^2, \tilde{m}_{Y\overline{\nu}}^2, \tilde{m}_{\nu\overline{\nu}}^2\right)$ as Cartesian coordinates.

- $\because \tilde{m}_{Y\nu}^2, \tilde{m}_{Y\overline{\nu}}^2, \tilde{m}_{\nu\overline{\nu}}^2$ are Lorentz scalars, the ‘effective’ Dalitz plot can be constructed in *any frame of reference*.

‘Effective’ Daitz plot method

Our tool for investigating the Majorana neutrinos is the ‘effective’ Dalitz plot.

Any point inside the ternary plot can be described by either polar coordinates (r,θ) or rectangular coordinates (x,y).



$$\tilde{m}^2_{\nu\overline{\nu}} = \frac{1}{3}(1 + r\ \cos\theta) = \frac{1}{3}(1+y),$$

$$\tilde{m}^2_{Y\nu} = \frac{1}{3}\left(1 + r\ \cos\left(\frac{2\pi}{3} + \theta\right)\right) = \frac{1}{6}\left(2 + \sqrt{3}x - y\right),$$

$$\tilde{m}^2_{Y\overline{\nu}} = \frac{1}{3}\left(1 + r\ \cos\left(\frac{2\pi}{3} - \theta\right)\right) = \frac{1}{6}\left(2 - \sqrt{3}x - y\right).$$

$$\nu \longleftrightarrow \overline{\nu} \equiv \tilde{m}^2_{Y\nu} \longleftrightarrow \tilde{m}^2_{Y\overline{\nu}} \equiv \theta_{\text{GJ}} \longleftrightarrow \pi + \theta_{\text{GJ}} \equiv \theta \longleftrightarrow -\theta.$$

NOTE: θ_{GJ} is an angle in the Gottfried-Jackson frame, however θ is the polar angle in the ‘effective’ Dalitz plot.

‘Effective’ Daitz plot method

We analyse the distribution of events in the ‘effective’ Dalitz plot to distinguish Dirac and Majorana neutrinos.

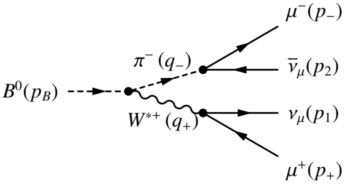
- The pattern of distribution of events in the ‘effective’ Dalitz plot is a consequence of *dynamics*.
- The dynamics is encoded in the *transition amplitude*.
- The amplitude for all the processes under our consideration, should be *anti-symmetrized for Majorana neutrinos*, while for Dirac case there is no such anti-symmetrization.
- The distribution of events should be completely symmetric under exchange of ν and $\overline{\nu}$ for Majorana neutrinos. For Dirac neutrinos the distribution must have some asymmetry under the above exchange.
- We shall mathematically show these assertions by considering one example decay explicitly.

'Effective' Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Dirac case:

- Feynman diagram



- Amplitude

$$\mathscr{M}^D \propto (p_- + p_2)_\alpha \left(F_1\,(p_- + p_2)_\beta + F_2\,(p_+ + p_1)_\beta\right)$$
$$\times \left[\overline{\psi}_{\mu^-}(p_-)\,\gamma^\alpha\left(1-\gamma^5\right)\psi_{\overline{\nu}}(p_2)\right]\left[\overline{\psi}_\nu(p_1)\,\gamma^\beta\left(1-\gamma^5\right)\psi_{\mu^+}(p_+)\right],$$

where F_1 and F_2 are form factors related to $B \to \pi$ transition, and can be expressed in terms of the usual form factors f_+ and f_0:

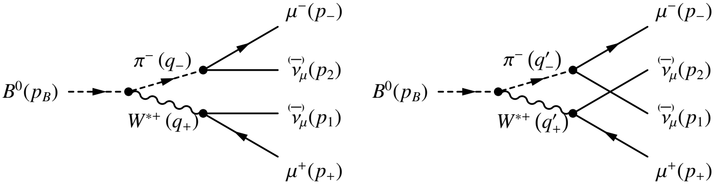
$$F_1 = f_+, \qquad F_2 = -\frac{m_B^2 - m_\pi^2}{(p_B - q_-)^2}\left(f_+ - f_0\right).$$

‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Majorana case:

- Feynman diagram



- Amplitude (antisymmetric under $p_1 \longleftrightarrow p_2$ exchange)

$$\begin{aligned}
\mathscr{M}^M \propto \Big(& (p_- + p_2)_\alpha \left(F_1\,(p_- + p_2)_\beta + F_2\,(p_+ + p_1)_\beta\right) \\
& \times \left[\overline{\psi}_{\mu^-}(p_-)\,\gamma^\alpha\left(1-\gamma^5\right)\psi_{\overline{\nu}}(p_2)\right]\left[\overline{\psi}_\nu(p_1)\,\gamma^\beta\left(1-\gamma^5\right)\psi_{\mu^+}(p_+)\right] \\
& - (p_- + p_1)_\alpha \left(F_1\,(p_- + p_1)_\beta + F_2\,(p_+ + p_2)_\beta\right) \\
& \times \left[\overline{\psi}_{\mu^-}(p_-)\,\gamma^\alpha\left(1-\gamma^5\right)\psi_{\overline{\nu}}(p_1)\right]\left[\overline{\psi}_\nu(p_2)\,\gamma^\beta\left(1-\gamma^5\right)\psi_{\mu^+}(p_+)\right]\Big).
\end{aligned}$$

‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Taking square of the amplitudes, then summing over the final spins and retaining only those terms that are independent of m_ν (for simplicity and also since m_ν is very small) we get,

$$\begin{aligned}
\left\langle\left|\mathscr{M}^D\right|^2\right\rangle \propto 64m_\mu^2\,(p_-\cdot p_2)\bigg(&|F_1|^2\left(2\,(p_+\cdot p_-+p_+\cdot p_2)\,(p_-\cdot p_1+p_{\overline{\nu}}\cdot p_1)-(p_+\cdot p_1)\left(m_\mu^2+2p_-\cdot p_2\right)\right)\\
&+|F_2|^2\,m_\mu^2\,(p_+\cdot p_1)+2\mathrm{Re}\left(F_1F_2^*\right)(p_-\cdot p_1+p_1\cdot p_2)\,m_\mu^2\bigg),\\
\left\langle\left|\mathscr{M}^M\right|^2\right\rangle \propto 64m_\mu^2\bigg(&(p_-\cdot p_2)\bigg(|F_1|^2\left(2\,(p_+\cdot p_-+p_+\cdot p_2)\,(p_-\cdot p_1+p_1\cdot p_2)-(p_+\cdot p_1)\left(m_\mu^2+2p_-\cdot p_2\right)\right)\\
&+|F_2|^2\,m_\mu^2\,(p_+\cdot p_1)+2\mathrm{Re}\left(F_1F_2^*\right)\left((p_-\cdot p_1+p_1\cdot p_2)\,m_\mu^2\right)\bigg)\\
&+(p_-\cdot p_1)\bigg(|F_1|^2\left(2\,(p_+\cdot p_-+p_+\cdot p_1)\,(p_-\cdot p_2+p_1\cdot p_2)-(p_+\cdot p_2)\left(m_\mu^2+2p_-\cdot p_1\right)\right)\\
&+|F_2|^2\,m_\mu^2\,(p_+\cdot p_2)+2\mathrm{Re}\left(F_1F_2^*\right)\left((p_-\cdot p_2+p_1\cdot p_2)\,m_\mu^2\right)\bigg)\bigg),
\end{aligned}$$

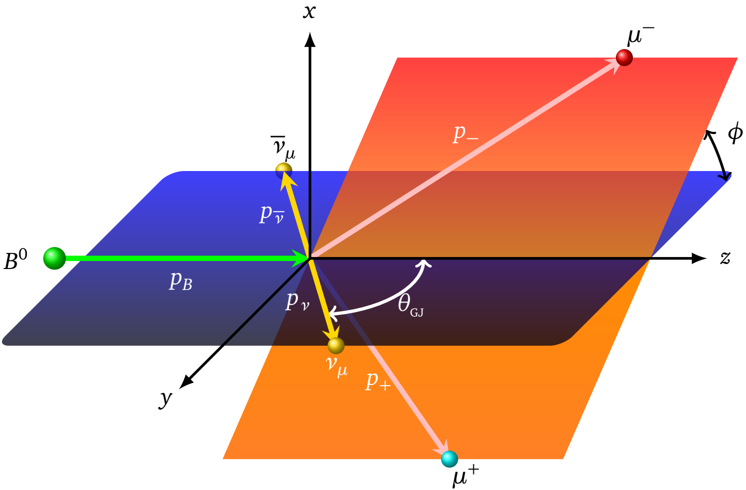
where m_μ is the mass of muon.

To evaluate the dot products we choose a particular frame of reference.

'Effective' Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Kinematics in the center-of-momentum frame of $\nu_\mu\overline{\nu}_\mu$



‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv Y\left[\mu^+\mu^-\right] \nu_\mu \overline{\nu}_\mu$.

The normalized angular distribution for the example decay mode has the following form,

- for Dirac case, ▸ Full distribution for Dirac case is here

$$\frac{1}{\Gamma^D} \frac{d^2\Gamma^D}{d\phi \, d\cos\theta_{\text{GJ}}} = T_0^D + T_1^D \cos\theta_{\text{GJ}} + T_2^D \cos^2\theta_{\text{GJ}} + \left(U_1^D \sin\theta_{\text{GJ}} + U_2^D \sin 2\theta_{\text{GJ}}\right) \cos\phi + V^D \sin^2\theta_{\text{GJ}} \cos^2\phi,$$

  where T_0^D, T_1^D, T_2^D, U_1^D, U_2^D and V^D are the angular coefficients, terms in red are odd under $\nu \longleftrightarrow \overline{\nu} \equiv \theta_{\text{GJ}} \longleftrightarrow \pi + \theta_{\text{GJ}}$ exchange, and

- for Majorana case, ▸ Full distribution for Majorana case is here

$$\frac{1}{\Gamma^M} \frac{d^2\Gamma^M}{d\phi \, d\cos\theta_{\text{GJ}}} = T_0^M + T_2^M \cos^2\theta_{\text{GJ}} + U_2^M \sin 2\theta_{\text{GJ}} \cos\phi + V^M \sin^2\theta_{\text{GJ}} \cos^2\phi,$$

  where T_0^M, T_2^M, U_2^M and V^M are the angular coefficients.

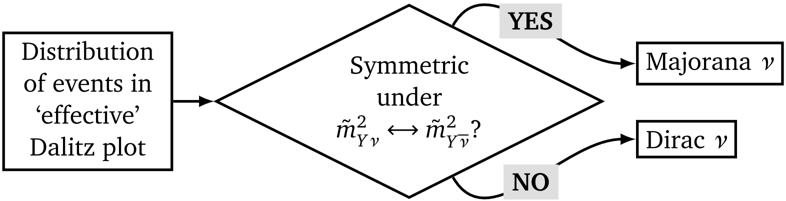
‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$.

- By looking at *angular distribution in the* $\cos\theta_{\text{GJ}}$-ϕ *plane*, we can distinguish the Dirac and Majorana cases.

- Since $\nu \longleftrightarrow \overline{\nu} \equiv \tilde{m}^2_{Y\nu} \longleftrightarrow \tilde{m}^2_{Y\overline{\nu}} \equiv \theta_{\text{GJ}} \longleftrightarrow \pi + \theta_{\text{GJ}} \equiv \theta \longleftrightarrow -\theta$, an asymmetry under $\theta_{\text{GJ}} \longleftrightarrow \pi + \theta_{\text{GJ}}$ will also give rise to an asymmetry under $\theta \longleftrightarrow -\theta$.

- **Signature of Majorana neutrinos:**



'Effective' Daitz plot method

The distribution of events in the 'effective' Dalitz plot can be described by a Fourier decomposition.

- Let $\mathscr{D}(r,\theta)$ denote the distribution of events inside the 'effective' Dalitz plot. Then,
  - $\mathscr{D}_D(r,\theta) = \sum_{n=0}^{\infty} \left(S_n^D(r)\ \sin(n\theta) + C_n^D(r)\ \cos(n\theta) \right)$ (Dirac neutrinos)
  - $\mathscr{D}_M(r,\theta) = \sum_{n=0}^{\infty} C_n^M(r)\ \cos(n\theta)$ (Majorana neutrinos)

  where $S_n^D(r)$ and $C_n^{D,M}(r)$ are the Fourier coefficients which are some functions of masses and energies of the particles involved.

‘Effective’ Daitz plot method

The Dirac and Majorana neutrinos leave two distinct signatures in the ‘effective’ Dalitz plot.

- **Signature of Majorana neutrinos:**
  - $\int dr\ \mathscr{D}_M(r,\theta) = \int dr\ \mathscr{D}_M(r,-\theta),$ (Majorana neutrinos)
  - $\int dr\ \mathscr{D}_D(r,\theta) \neq \int dr\ \mathscr{D}_D(r,-\theta),$ (Dirac neutrinos)

  where we have carried out integrations radially, i.e. we add all the events inside the ‘effective’ Dalitz plot along the radial direction at any chosen polar angle.

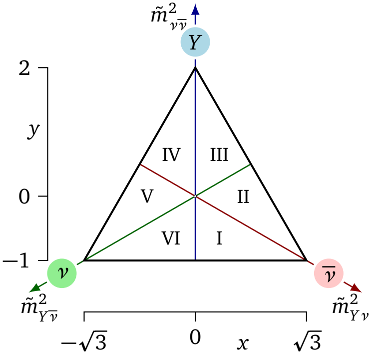
- This distinction between Dirac and Majorana neutrinos is always present in our ‘effective’ Dalitz plot *irrespective of neutrino mass*.

- The distribution asymmetry inside ‘effective’ Dalitz plot can be quantified by some asymmetries.

‘Effective’ Daitz plot method

The signature of Majorana neutrinos can be quantified in terms of some easily observable asymmetries.

- **Sextant asymmetry:**



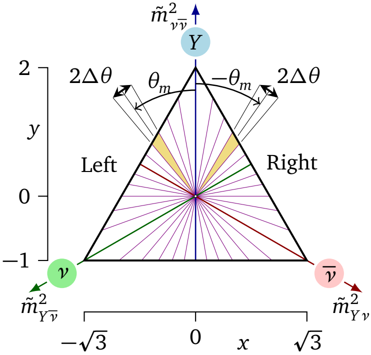
$$\mathscr{A} = \left|\frac{N_I - N_{VI}}{N_I + N_{VI}}\right| + \left|\frac{N_{II} - N_V}{N_{II} + N_V}\right| + \left|\frac{N_{III} - N_{IV}}{N_{III} + N_{IV}}\right|,$$

where N_i denotes the number of events in the ith sextant.

‘Effective’ Daitz plot method

The signature of Majorana neutrinos can be quantified in terms of some easily observable asymmetries.

❖ **Binned asymmetry:**



$$\mathscr{A}' = \sum_{\theta_m} \left| \frac{N(\theta_m) - N(-\theta_m)}{N(\theta_m) + N(-\theta_m)} \right|,$$

where $N(\theta_m)$ is the number of events in an angular bin $\theta_m \pm \Delta\theta$.

‘Effective’ Daitz plot method

There exist a plethora of processes which can be looked at using our approach.

Following is a tentative list of processes that can be studied using our approach for deciphering the Majorana nature of neutrinos.

X	$\rightarrow$	intermediate resonances	$\rightarrow$	final state $(Y)\,\nu_\ell\,\overline{\nu}_\ell$
B^0		$D^-\ell^+\nu_\ell$		$\left(K^0\ell^+\ell^-\right)\nu_\ell\overline{\nu}_\ell$
B^+		$\bar{D}^0\ell^+\nu_\ell$		$(K^-\ell^+\ell^-)\,\nu_\ell\overline{\nu}_\ell$
B^0, D^0, K^0		$\pi^\pm\ell^\mp\overset{(-)}{\nu}_\ell$		$(\ell^+\ell^-)\,\nu_\ell\overline{\nu}_\ell$
		$\pi^+\pi^-$		$(\ell^+\ell^-)\,\nu_\ell\overline{\nu}_\ell$
$\overline{B}^0, B_s^0, D^0$		π^+K^-		$(\ell^+\ell^-)\nu_\ell\overline{\nu}_\ell$
B_s^0		$K^-\ell^+\nu_\ell$		$(\ell^+\ell^-)\nu_\ell\overline{\nu}_\ell$
K_S^0		$\pi^+\pi^-\gamma$		$(\ell^+\ell^-\gamma)\nu_\ell\overline{\nu}_\ell$
B^0, D^0, $K_L^0, J/\psi(1S)$		$\pi^+\pi^-\pi^0$		$(\ell^+\ell^-\pi^0)\nu_\ell\overline{\nu}_\ell$

‘Effective’ Daitz plot method

There exist a plethora of processes which can be looked at using our approach.

X $\rightarrow$	intermediate resonances $\rightarrow$	final state $(Y)\nu_\ell\overline{\nu}_\ell$
D^0	$\pi^+\pi^-K^0_S$	$(\ell^+\ell^-K^0_S)\nu_\ell\overline{\nu}_\ell$
	$\pi^+K^-\pi^0$	$(\ell^+\ell^-\pi^0)\nu_\ell\overline{\nu}_\ell$
	$K^+K^-K^0_S$	$(\ell^+\ell^-K^0_S)\nu_\ell\overline{\nu}_\ell$
	$\pi^+\pi^-2\pi^0$	$(\ell^+\ell^-2\pi^0)\nu_\ell\overline{\nu}_\ell$
	$K^-\ell^+\nu_\ell$	$(\ell^+\ell^-)\,\nu_\ell\overline{\nu}_\ell$
D^+	$\bar{K}^0\ell^+\nu_\ell$	$(\pi^+\ell^-\ell^+)\,\nu_\ell\overline{\nu}_\ell$
$J/\psi(1S)$	$\pi^+\pi^-\omega$	$(\ell^+\ell^-\omega)\nu_\ell\overline{\nu}_\ell$
	$\pi^+\pi^-\eta$	$(\ell^+\ell^-\eta)\nu_\ell\overline{\nu}_\ell$
	$\pi^+\pi^-\phi$	$(\ell^+\ell^-\phi)\nu_\ell\overline{\nu}_\ell$
	$\pi^+\pi^-\omega\pi^0$	$(\ell^+\ell^-\omega\pi^0)\nu_\ell\overline{\nu}_\ell$
$\Upsilon(2S)$	$\pi^+\pi^-\Upsilon(1S)$	$(\ell^+\ell^-\Upsilon(1S))\nu_\ell\overline{\nu}_\ell$

Results from numerical simulation

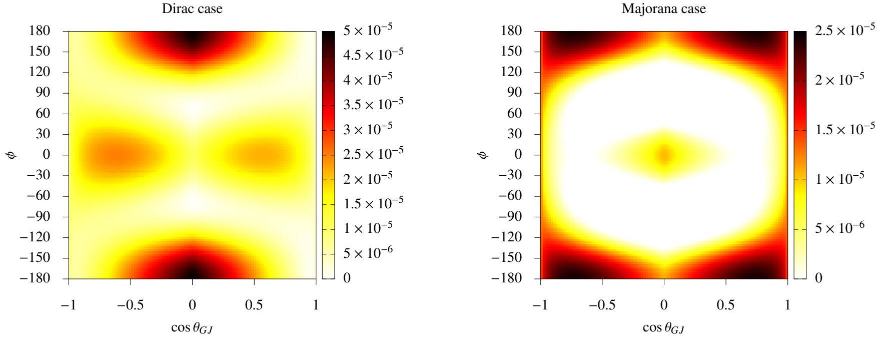
- To demonstrate the usefulness of our proposed method we have carried out numerical simulations for the following processes,
  - $X\left[B^0\right] \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$,
  - $X\left[B_s^0\right] \to K^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$,
  - $X\left[B^0\right] \to \pi^+ \left(\to \mu^+ \nu_\mu\right) \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$.

- For each process we have simulated 10^5 events while neglecting the mass of neutrino in comparison with other masses in the processes, and the resulting scatter plots for both angular distribution and 'effective' Dalitz plot are fitted with the functional dependencies taken directly from theoretical results.

Results from numerical simulation

For the decay $B^0 \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

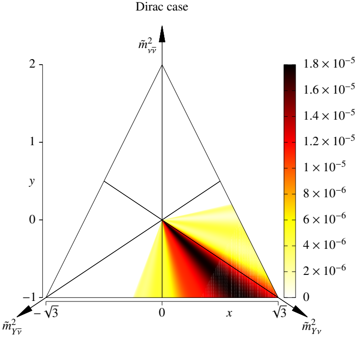
Comparison of best fit normalized angular distribution $\dfrac{1}{\Gamma} \dfrac{d^2\Gamma}{d\phi \; d\cos\theta_{\text{GJ}}}$

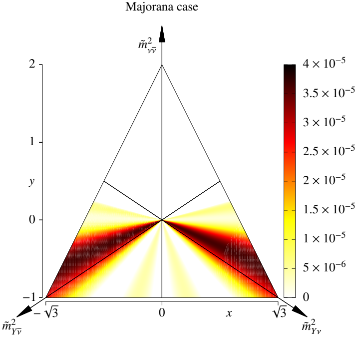


Results from numerical simulation

For the decay $B^0 \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

Comparison of best fit distribution of events
inside the 'effective' Dalitz plot
for Dirac and Majorana cases

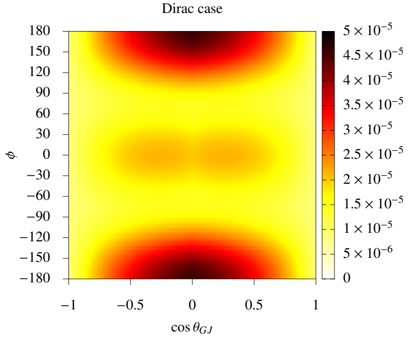


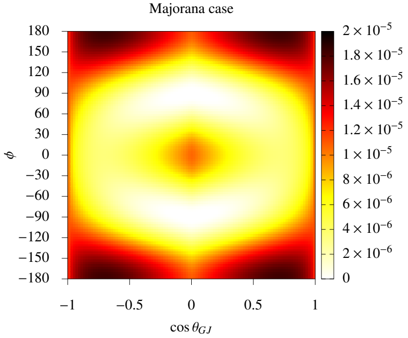


Results from numerical simulation

For the decay $B_s^0 \to K^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

Comparison of best fit normalized angular distribution $\dfrac{1}{\Gamma} \dfrac{d^2\Gamma}{d\phi \; d\cos\theta_{\mathrm{GJ}}}$





Results from numerical simulation

For the decay $B_s^0 \to K^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

Comparison of best fit distribution of events
inside the 'effective' Dalitz plot
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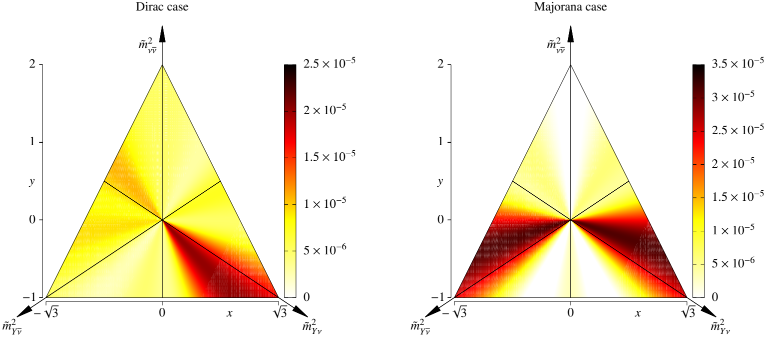
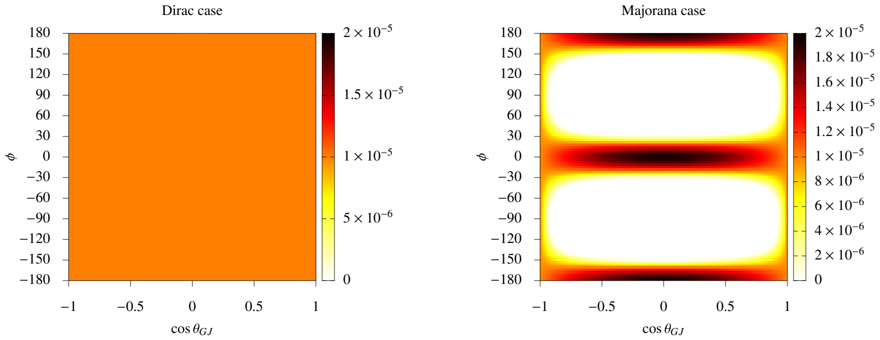


Results from numerical simulation

For the decay $B^0 \to \pi^+ (\to \mu^+ \nu_\mu)\, \pi^- (\to \mu^- \overline{\nu}_\mu) \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

Comparison of best fit normalized angular distribution $\dfrac{1}{\Gamma} \dfrac{d^2\Gamma}{d\phi\, d\cos\theta_{\text{GJ}}}$

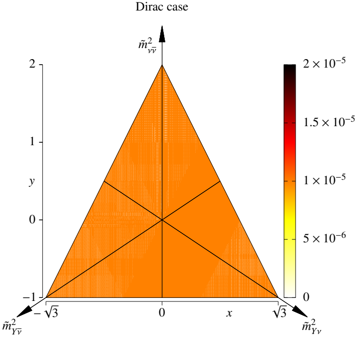


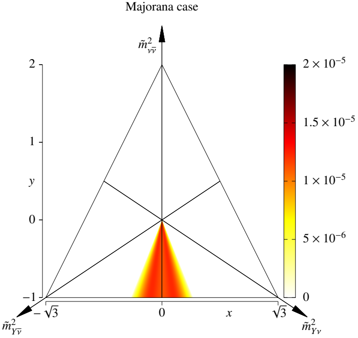
The flat distribution for Dirac case here is accidental. Details are shown here

Results from numerical simulation

For the decay $B^0 \to \pi^+ \left(\to \mu^+ \nu_\mu\right) \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

Comparison of best fit distribution of events
inside the 'effective' Dalitz plot
for Dirac and Majorana cases





Conclusion

Salient features of our methodology

- Processes are *not rare* for our case, unlike $0\nu\beta\beta$ and other $\Delta L = 2$ processes.

- Majorana and Dirac neutrinos have completely *distinct signatures*, which survive even when one considers neutrinos to be almost massless.

- The signatures are quantifiable by easily *observable asymmetries* defined on 'effective' Dalitz plots. For $m_\nu \to 0$, rate of $\Delta L = 2$ processes $\to 0$, but our asymmetries $\not\to 0$.

- Since our kinematical tests *directly probe* the quantum statistical nature of Majorana neutrino and anti-neutrino, they remain unaffected by the practical Dirac-Majorana confusion theorem.

Conclusion

By analysing the quantum statistical property of Majorana neutrino and anti-neutrino via ‘effective’ Dalitz plots for suitably well choosen processes we can look for the Majorana nature of active sub-eV neutrinos.

Thank You

Back-up slides

Angular distribution

For the decay $B^0 \to \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \mu^+ \nu_\mu \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$: Dirac case

The full angular distribution for Dirac case is

$$
\begin{aligned}
\frac{d\Gamma^D}{d_4\mathrm{PS}} \propto 8m_\mu^2 \Bigg(& 4m_\mu^2 \mathrm{Re}\left(F_1 F_2^*\right) \Big(4\left(EE'\left(E^2 + EE' + p^2\right) - pp'\left(E^2 + p^2\right)\sin\theta_{\text{GJ}}\cos\phi - p^2 p'^2 \sin^2\theta_{\text{GJ}}\cos^2\phi\right) \\
& \quad + 2\,|\vec{p}_B|\,p\cos\theta_{\text{GJ}}\left(E^2 + p^2 + 2pp'\sin\theta_{\text{GJ}}\cos\phi\right) - |\vec{p}_B|^2 p^2\cos^2\theta_{\text{GJ}} \Big) \\
& + |F_1|^2\left(2EE' + |\vec{p}_B|\,p\cos\theta_{\text{GJ}} - 2pp'\sin\theta_{\text{GJ}}\cos\phi\right)\Big(8E^3E' \\
& \quad + 2pp'\sin\theta_{\text{GJ}}\cos\phi\left(4E^2 + 16EE' + 4E'^2 + 2m_\mu^2 - |\vec{p}_B|^2 + 4p^2 + 4p'^2\right) \\
& \quad + |\vec{p}_B|\,p\cos\theta_{\text{GJ}}\left(4E^2 - 4E'^2 + 2m_\mu^2 + |\vec{p}_B|^2 + 4p^2 - 4p'^2\right) \\
& \quad + 8E^2E'^2 - 2E^2\,|\vec{p}_B|^2 + 8E^2p'^2 + 8EE'^3 - 4EE'm_\mu^2 - 2EE'\,|\vec{p}_B|^2 \\
& \quad + 8EE'p^2 + 8EE'p'^2 + 8E'^2p^2 - 2\,|\vec{p}_B|^2 p^2 + 8p^2p'^2\Big) \\
& + 2\,|F_2|^2 m_\mu^2\left(4\left(EE' - pp'\sin\theta_{\text{GJ}}\cos\phi\right)^2 - |\vec{p}_B|^2 p^2\cos^2\theta_{\text{GJ}}\right) \Bigg),
\end{aligned}
$$

where $d_4\mathrm{PS}$ denotes the differential 4-body phase space, E and E' are the energies of ν_μ (or $\overline{\nu}_\mu$) and muon ($\mu^\pm$) respectively, p is the magnitude of the 3-momentum of ν_μ (or $\overline{\nu}_\mu$), p' is the magnitude of the projection of 3-momentum of $\mu^\pm$ on the xy-plane. Simplified distribution for Dirac case is here

Angular distribution

For the decay $B^0 \to \pi^-\left(\to \mu^-\overline{\nu}_\mu\right)\mu^+\nu_\mu \equiv \mu^+\mu^-\nu_\mu\overline{\nu}_\mu$: Majorana case

The full angular distribution for Majorana case is

$$\begin{aligned}
\frac{d\Gamma^M}{d_4\mathrm{PS}} \propto 32m_\mu^2\Bigg(&\frac{1}{2}\,|F_2|^2\,m_\mu^2\left(4E^2E'^2+4p^2p'^2\sin^2\theta_{\text{GJ}}\cos^2\phi-|\vec{p}_B|^2p^2\cos^2\theta_{\text{GJ}}\right)\\
&+|F_1|^2\Bigg(\frac{1}{4}p^2\Big(-4p'^2\sin^2\theta_{\text{GJ}}\cos^2\phi\left(4\left(E^2+4EE'+E'^2\right)+2m_\mu^2-|\vec{p}_B|^2+4\left(p^2+p'^2\right)\right)\\
&+|\vec{p}_B|^2\cos^2\theta_{\text{GJ}}\left(4E^2-4E'^2+2m_\mu^2+|\vec{p}_B|^2+4p^2-4p'^2\right)\\
&+4\,|\vec{p}_B|\,p'\sin\theta_{\text{GJ}}\cos\theta_{\text{GJ}}\cos\phi\left(4E'(2E+E')-|\vec{p}_B|^2+4p'^2\right)\Big)\\
&+EE'\Big(4E^3E'+E^2\left(4E'^2-|\vec{p}_B|^2+4p'^2\right)\\
&+EE'\left(4E'^2-2m_\mu^2-|\vec{p}_B|^2+4\left(p^2+p'^2\right)\right)+p^2\left(4E'^2-|\vec{p}_B|^2+4p'^2\right)\Big)\Bigg)\\
&+m_\mu^2\mathrm{Re}\left(F_1F_2^*\right)\left(4EE'\left(E(E+E')+p^2\right)-(|\vec{p}_B|\,p\cos\theta_{\text{GJ}}-2pp'\sin\theta_{\text{GJ}}\cos\phi)^2\right)\Bigg).
\end{aligned}$$

◂ Simplified distribution for Majorana case is here

Angular distribution

For the decay $B_s^0 \to \pi^- (\to \mu^- \overline{\nu}_\mu) \pi^+ (\to \mu^+ \nu_\mu) \equiv \mu^+ \mu^- \nu_\mu \overline{\nu}_\mu$

- **Dirac case:**

$$\frac{d\Gamma^D}{d_4\mathrm{PS}} \propto 64 m_\mu^4 \Big(E^2 E'^2 + p^2 p'^2 \cos^2\phi \sin^2\theta_{\mathrm{GJ}} - \frac{1}{4} |\vec{p}_B|^2 p^2 \cos^2\theta_{\mathrm{GJ}} - 2EE'pp' \cos\phi \sin\theta_{\mathrm{GJ}} \Big)$$
$$= 64 m_\mu^4 \left(m_\pi^2 - m_\mu^2 \right)^2 / 4.$$

- **Majorana case:**

$$\frac{d\Gamma^M}{d_4\mathrm{PS}} \propto 64 m_\mu^4 \Big(E^2 E'^2 + p^2 p'^2 \cos^2\phi \sin^2\theta_{\mathrm{GJ}} - \frac{1}{4} |\vec{p}_B|^2 p^2 \cos^2\theta_{\mathrm{GJ}} \Big).$$

NOTE: This case has accidental symmetry for the Dirac case under $\nu \longleftrightarrow \overline{\nu}$ exchange. The Dirac case is fully flat. Nevertheless, by angular distribution both Dirac and Majorana cases can be distinguished.

Angular distribution plots are shown here