Almost One-hundred year's QUEST :

1930 ~

Are (active sub-eV) neutrinos Dirac or Majorana ?

$\nu \neq \nu^c$

Dirac particle

ν

Majorana particle

$\nu = \nu^c = C\overline{\nu}^T$

practical Dirac-Majorana Confusion Theorem for sub-eV active neutrino, and How to overcome it?

NCTS-NTU Taiwan 03/05/2025
Academia Sinica 02/26/2025

KMI, Nagoya Univ. 07/16/2024
UW Wisc-Pheno, 08/23/2023
DPF, Mexico Physics Society, 06/12/2023
SNU-Physics, 03/30/2023
MPI-Heidelberg, Germany, 01/23/2023
High Energy Physics in LHC Era, Chile, 01/10/2023
Apres-LHC Workshop, 08/09/2022
4th Mini WS on CUBES, 11/05/2022
WS on Physics of Dark Cosmos, 10/23/2022
KEK-IPNS, Japan, 08/22/2022
IHEP, China, 07/15/2022

C S Kim (w/ M V N Murthy, J Rosiek, D Sahoo, K Vishnudath)
Yonsei University: ChonNam National University-SRC

CSK,MM,DS, PRD105 (2022)
CSK,JR,DS, EPJC83 (2023)
CSK, EPJC83 (2023)
CSK,DS,KV, EPJC84(2024)

(Is the sub-eV active neutrino Dirac or Majorana ?)
Alternative to $0\nu\beta\beta$:

Quantum Statics + $2\nu\beta\beta$!!

UW-Madison 06/27/2025
FNAL 06/20/2025
CNU-SRC 05/16/2025
CUBES-7 04/26/2025

W/ D. Sahoo, Meng-Ru Wu

Choong Sun Kim
Yonsei University: CNU-SRC

CSK,MM,DS, PRD105 (2022); 2106.11785
CSK,JR,DS, EPJC83 (2023); 2209.10110
CSK, EPJC83 (2023); 2307.05654
CSK, DS, KV, EPJC84 (2024); 2405.17341

CONTENTS

- Introduction (nu Mass, Seesaw & 0nuBB)
- practical Dirac-Majorana Confusion Theorem (pDMCT)
- How to overcome pDMCT ?
- Alternatives to $0\nu\beta\beta$
- Back-to-back $\nu - \bar{\nu}$ in $2\nu\beta^{+}\beta^{-}$
- Summary

INTRODUCTION

Sub-eV active neutrino mass

Seesaw mechanism

$\Delta L = 2$ processes & 0-nu-Beta-Beta

(sub-eV active) neutrinos have mass

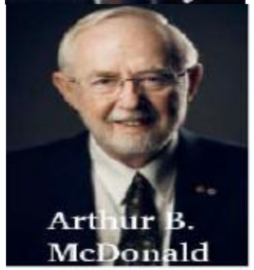
- Neutrinos are massless in SM, $m_\nu = 0$.
  All neutrinos are only left-handed (ν_L).

$$\mathscr{L}^D_{\text{mass}} = -m_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R\right), \qquad m_\nu = \frac{Y_\nu\, \mathrm{v}}{\sqrt{2}},$$

  where Y_ν = Higgs-neutrino Yukawa coupling constant,
  and v = Higgs VEV.
  No way to generate mass without right-handed neutrinos (ν_R).

- But observations of neutrino oscillation imply that **neutrinos have mass**, $m_\nu \neq 0$.

  The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (Sudbury Neutrino Observatory) "for the discovery of neutrino oscillations, which shows that neutrinos have mass".



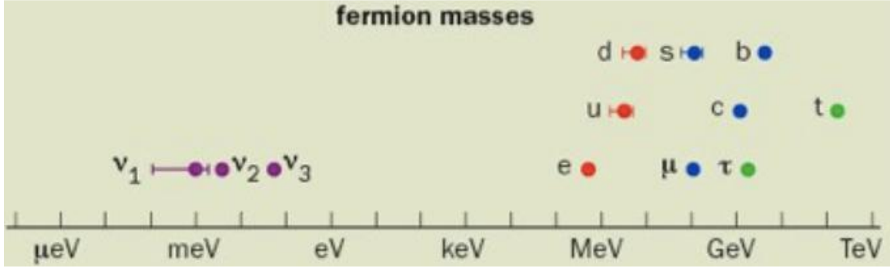
How to give neutrinos mass?

There are various suggestions as to how neutrinos can get mass.

- **Dirac mass:**
  - *Assumption:* ν_R exists.
  - *Lagrangian:*

$$\mathscr{L}^D_{\text{mass}} = -m^D_\nu \left(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R \right).$$

  - *Disadvantage:* No reason for m^D_ν to be small.



- **Majorana mass:**
  - *Assumption:* neutrino $\equiv$ anti-neutrino.
  - *Lagrangian:*

$$\mathscr{L}^M_{\text{mass}} = \frac{1}{2} m^M_\nu \left(\overline{\nu^C_L}\nu_L + \overline{\nu_L}\nu^C_L \right).$$

  - *Disadvantage:* $\mathscr{L}^M_{\text{mass}}$ is not invariant under $SU(2)_L \times U(1)_Y$ gauge group, so not allowed by SM.

How to give neutrinos (very small) mass ?

- **See-saw mechanism:** A simpler version of Dirac-Majorana mass, with a nice twist.

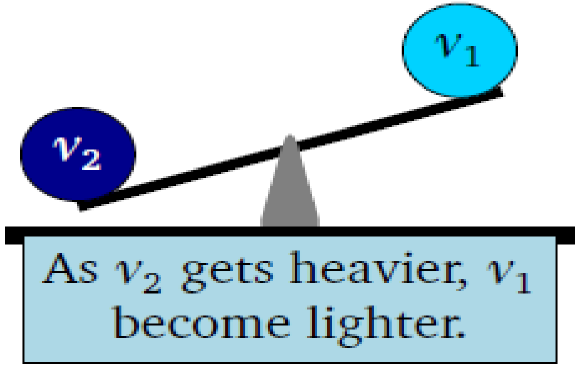
[PM,PLB67(1977)421]

  - *Assumptions:* $m_\nu^L = 0$ and $m_\nu^D \ll m_\nu^R$.
  - *Lagrangian:*
    $\mathscr{L}_{\text{mass}}^{D+M} = \frac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R \right) - m_\nu^D \left(\overline{\nu_R} \nu_L \right) + \text{H.c.} = \frac{1}{2} \overline{N_L^C} M N_L + \text{H.c.}$, where
    $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & m_\nu^R \end{pmatrix}$ is the mass matrix.
  - *Mass eigenvalues:*

$$m_{2,1} = \frac{1}{2} \left(m_\nu^R \pm \sqrt{\left(m_\nu^R \right)^2 + 4 \left(m_\nu^D \right)^2} \right)$$

$$\approx \frac{1}{2} m_\nu^R \left(1 \pm 1 \pm 2 \left(\frac{m_\nu^D}{m_\nu^R} \right)^2 \right).$$

$$\Longrightarrow m_1 \approx -\frac{\left(m_\nu^D \right)^2}{m_\nu^R} \text{ and } m_2 \approx m_\nu^R. \qquad \sin\theta \approx m_\nu^D / m_\nu^R$$



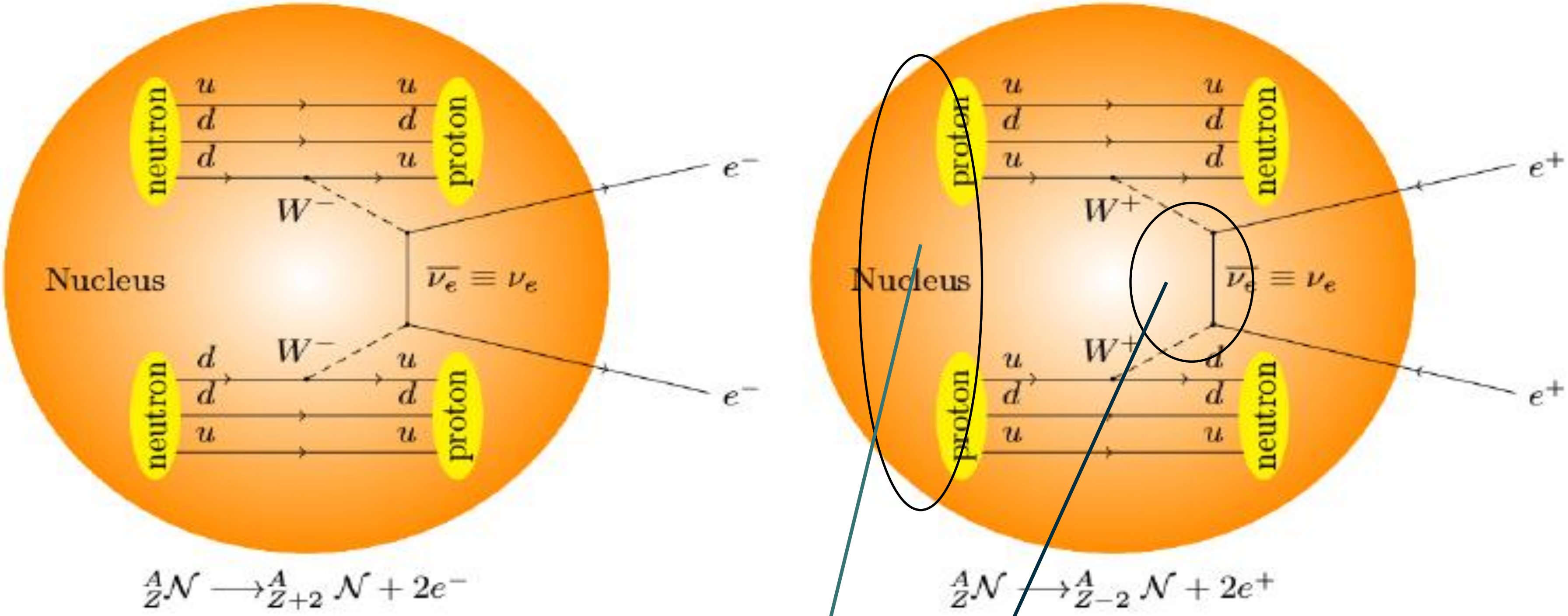
  - *Advantage:* $m_1 \ll m_2$, so light neutrinos are possible.
  - *Challenges:*
    - To find the heavy ν_2 experimentally.
    - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

6 / 1

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (1)

Lepton Number Violation (LNV)
➔ not allowed within SM

❖ Process:



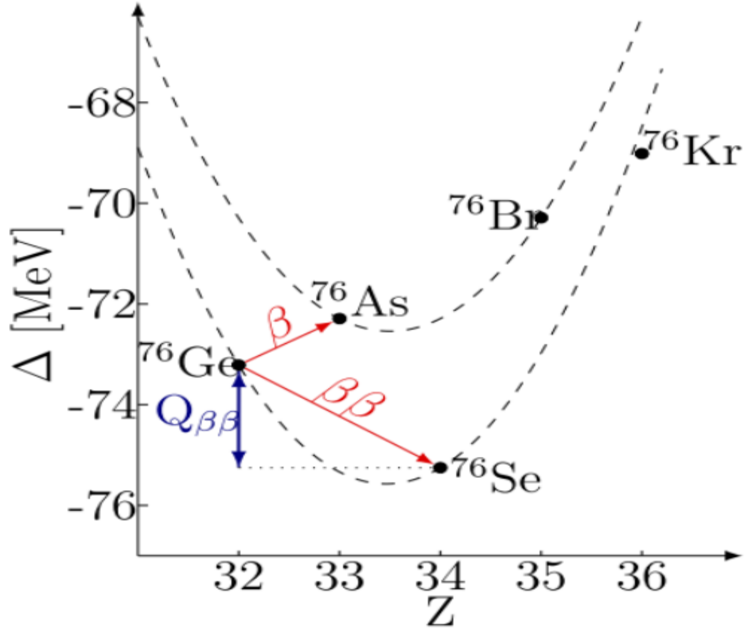
Doubly weak charged current process

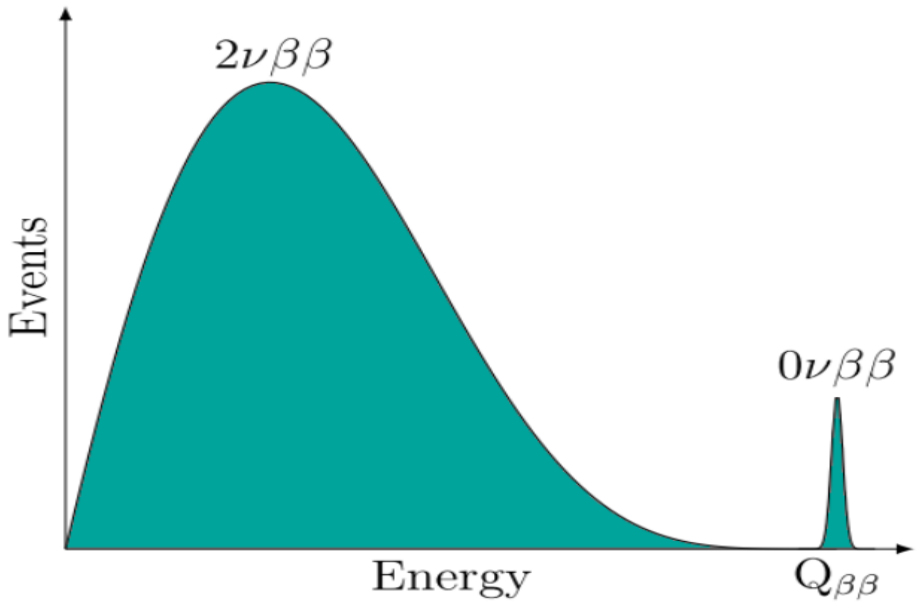
❖ The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\ \left|M_{0\nu}\right|\ \left|m_{\beta\beta}\right|^2,$$

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (2)

- Double-beta $(2\nu\beta\beta)$ decay has been observed in 10 isotopes, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, with half-life $T_{1/2} \approx 10^{18} - 10^{24}$ years.





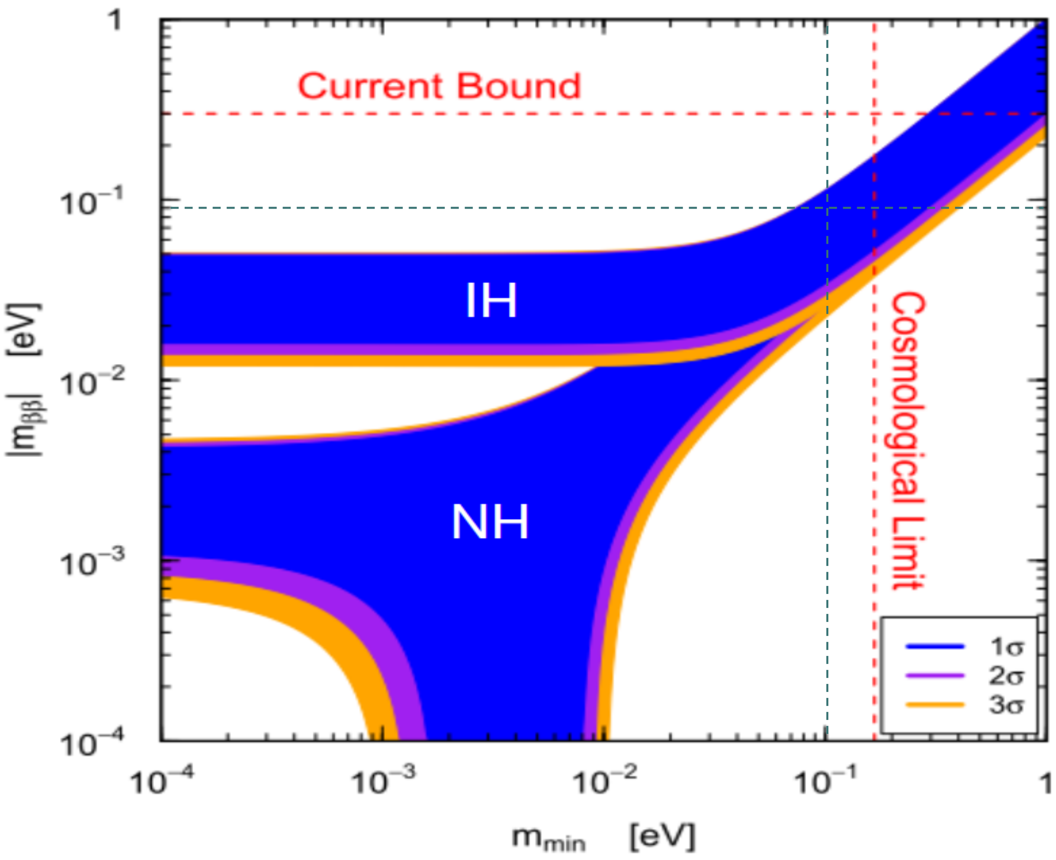
Giovanni Benato (for the GERDA collaboration), arXiv:1509.07792

- **$0\nu\beta\beta$ (forbidden in SM) is yet to be observed in any experiment.**

$$T^{0\nu}_{1/2}\left[{}^{76}\mathrm{Ge}\right] > 2.1\times 10^{25}\ \text{years}\,(90\%\ \text{C.L.})\,.$$

M. Agostini *et al.* (GERDA Collaboration) Phys. Rev. Lett. **111**, 122503 (2013).

Neutrino-less double-beta decay $(0\nu\beta\beta)$ (3)



NH: Normal hierarchy
IH: Inverted hierarchy

S. M. Bilenky and C. Giunti
Mod. Phys. Lett. A **27**, 1230015 (2012),
arXiv:1203.5250

- If $m_{\beta\beta} < 10^{-2}$, only NH is viable and the $T_{1/2}^{0\nu}$ will be much larger than the current experimental lower boun

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\ |M_{0\nu}|\ |m_{\beta\beta}|^2$$

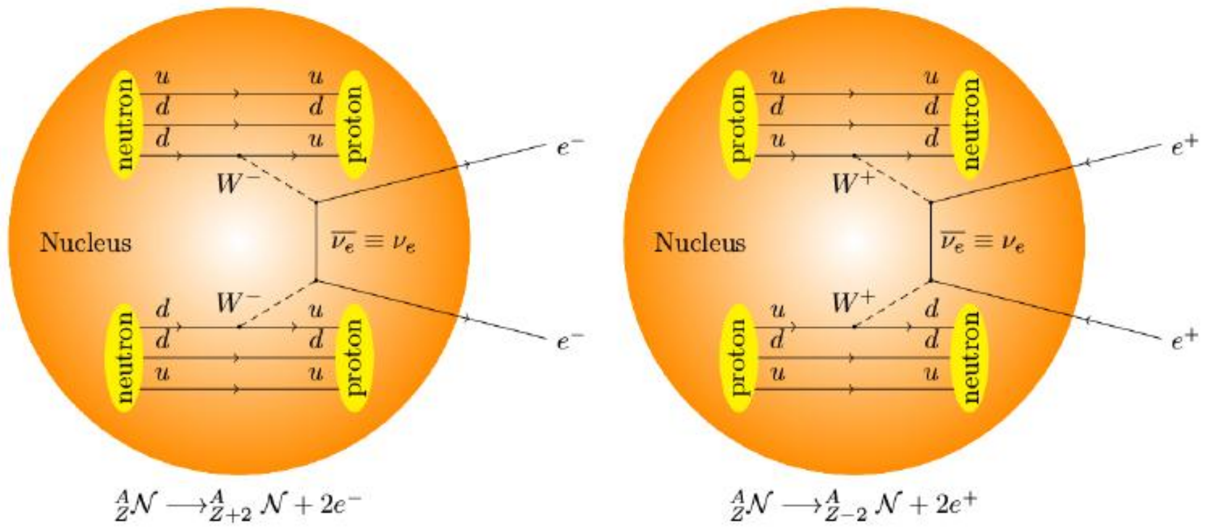
Neutrino-less double-beta decay $(0\nu\beta\beta)$ (4)

Is $0\nu\beta\beta$ the evidence for (sub-eV) active neutrino being Majorana fermion ?

➔ Evidence of Lepton Number Violation

➔ NOT unequivocal evidence of active (sub-eV) neutrino being Majorana neutrino !!

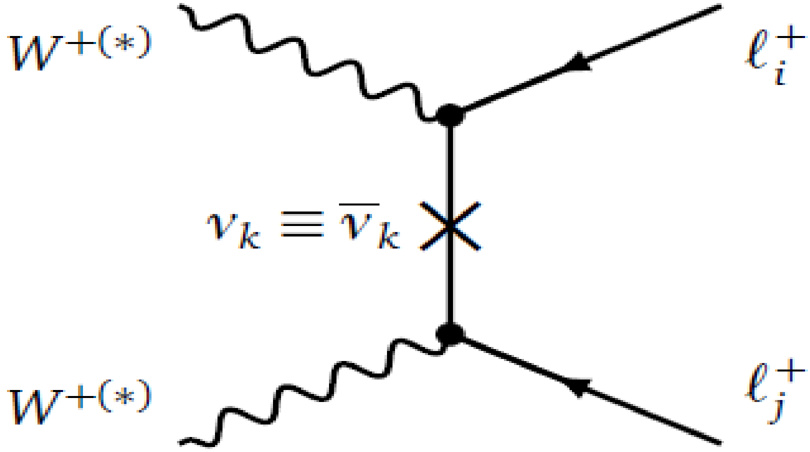
❖ Process:



e.g. heavy sterile neutrinos can give the same effects

Looking for Majorana neutrinos via $\Delta L = 2$ processes (1)

- Neutrinos are the only *elementary fermions* known to us that *can* have Majorana nature.
- Majorana neutrinos: $\nu \equiv \overline{\nu}$.
- Majorana neutrinos violate lepton flavor number (L), they mediate $\Delta L = 2$ processes.



$$\propto \int \frac{d^4p}{(2\pi)^4} \sum_k U_{\ell_i k}\, U_{\ell_j k}\, \frac{m_k + \not{p}}{p^2 - m_k^2}$$

- $\Delta L = 2$ processes play crucial rule to probe Majorana nature of ν's.
  - $m_k^2 \ll p^2$ — neutrinoless double-beta ($0\nu\beta\beta$) decay
  - $m_k^2 \approx p^2$ — Rare meson decays with $\Delta L = 2$
  - $m_k^2 \gg p^2$ — Collider searches at LHC

Looking for Majorana neutrinos via $\Delta L = 2$ processes (2)

❖ **Decay rate of any $\Delta L = 2$ process with final leptons $\ell_1^+ \ell_2^+$:**

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} \frac{m_k}{p^2 - m_k^2 + i m_k \Gamma_k} \right|^2 ,$$

where we have used the fact that $(1-\gamma^5)\, \not{p} (1-\gamma^5) = 0$.

- Light ν:

  $m_k^2 << p^2$

  $$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} m_k \right|^2 = |m_{\ell_1 \ell_2}|^2 .$$

  $|m_{\beta\beta}|^2 \sim (0.01\mathrm{eV})^2 = (10^{-22})\mathrm{GeV}^2$

- Heavy ν:

  $m_k^2 >> p^2$

  $$\Gamma_{\Delta L=2} \propto \left| \sum_k \frac{U_{\ell_1 k} U_{\ell_2 k}}{m_k} \right|^2 .$$

  $m_\nu^R \sim 10^5\,\mathrm{GeV}$ $\quad U \approx \frac{m_\nu^D}{m_\nu^R} \sim 10^{-3}$

- Resonant ν:

  $m_k^2 \approx p^2$

  $$\Gamma_{\Delta L=2} \propto \frac{\Gamma(N \to i)\, \Gamma(N \to f)}{m_N \Gamma_N} .$$

  $B^+ \to K^- \mu^+ \mu^+, \ldots$

[CS Kim etal, PRD82(2010)]

C.S. Kim, arXiv:2307.05654 [EPJC83(2023)]

practical Dirac-Majorana Confusion Theorem

practical Dirac-Majorana Confusion Theorem (pDMCT)

Discussions on pDMCT

practical Dirac-Majorana Confusion Theorem

The difference between Dirac and Majorana neutrinos
via any kinematical observable would be practically impossible to determine due to fact that the observable difference between Dirac and Majorana neutrinos is proportional to the tiny neutrino mass.

B. Kayser, PRD26, 1662 (1982)

(hidden/overlooked) Issues :

1. Historically, only SM allowed neutral current interaction mediated processes were analyzed.
2. No way to gain any information regarding individual neutrino antineutrino energies or 3-momenta. This invariably leads to integration over neutrino antineutrino related kinematic variables.

No model-independent, process-independent and observable-independent proof of this so-called "theorem". All processes where it was shown to hold involved full integration over the 4-momenta of missing neutrinos and/or only for Z^(*) → nu nubar case .

History trying to overcome DMCT, but only confirming

All for weak neutral current process in SM

$\gamma^* \to \nu\bar{\nu}$	[B Kayser, PRD26(1982)] ** [0]
$Z \to \nu\bar{\nu}$	[RE Shrock, eConf(1982)]
$e^+e^- \to \nu\bar{\nu}$	[E Ma, JT Pantaleone, PRD40(1989)]
$K^+ \to \pi^+\nu\bar{\nu}$	[JF Nieves, PB Pal, PRD32(1985)]
$e^+e^- \to \nu\bar{\nu}\gamma$	[T Chabra, PR Babu, PRD46(1992)] ** [2]
$\|es> \to \|gs> +\gamma\nu\bar{\nu}$	[Y Yoshimura, PRD75(2007)], ….
$e^-\gamma \to e^-\nu\bar{\nu}$	[JM Berryman etal, PRD98(2018)]

[1] S.P.Rosen,PRL48(1982); W.Rodejohann etal,JHEP05(2017).

** All practically impossible to measure momenta of nu-nubar ➔ Need integrate out ➔ pDMCT

practical Dirac-Majorana Confusion Theorem (1)

Consider the SM allowed decay, e.g.

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

Amplitude for Dirac case

$$\mathscr{M}^D = \mathscr{M}(p_1, p_2),$$

not helicity flip, but momentum exchange

For Majorana case

$$\mathscr{M}^M = \frac{1}{\sqrt{2}}\Big(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\Big).$$

Difference between D and M

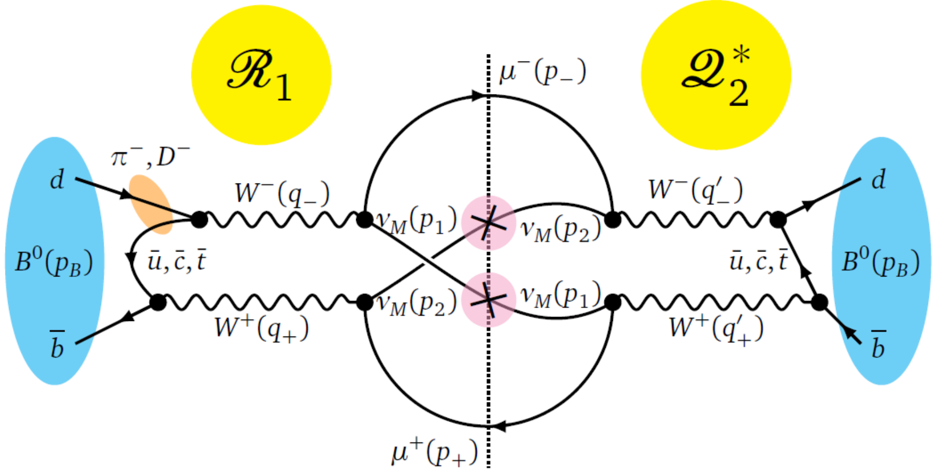
$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1, p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2, p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\Big(\mathscr{M}(p_1, p_2)^* \mathscr{M}(p_2, p_1)\Big)}_{\text{Interference term}}.$$

Dirac-Majorana Confusion Theorem (2)

Interference term

(within SM)

$$\mathrm{Re}\Big(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\Big) \propto m_\nu^2.$$



<u>In general</u>

$$\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}.$$

🡐 Possibly only source of D-M difference w/ SM

However, after integration

(required if momenta p_1 and p_2 are unobservable and missing)

$$\iint \underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} \, \mathrm{d}^4p_1 \, \mathrm{d}^4p_2 = \iint \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}} \, \mathrm{d}^4p_1 \, \mathrm{d}^4p_2,$$

Dirac-Majorana Confusion Theorem (2)

Interference term

$\mathscr{R}_1$ $\mathscr{Q}_2^*$

** Special case : $|M(p_1,p_2)|^2 = |M(p_2,p_1)|^2$

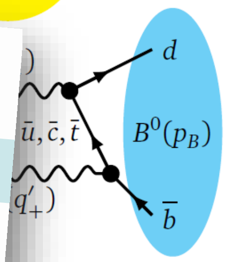
$$|\mathscr{M}^D|^2 - |\mathscr{M}^M|^2 = \frac{1}{2}\Big(\underbrace{|\mathscr{M}(p_1,p_2)|^2}_{\text{Direct term}} - \underbrace{|\mathscr{M}(p_2,p_1)|^2}_{\text{Exchange term}}\Big) + \underbrace{\mathrm{Re}\big(\mathscr{M}(p_1,p_2)^* \mathscr{M}(p_2,p_1)\big)}_{\text{Interference term}}. \longrightarrow m_\nu^2$$

<u>pDMCT holds at the Amplitude-Squared level</u>

➔ Difference between Dirac and Majorana neutrinos becomes 0 at every observable

➔ e.g. Within the SM, weak neutral current decays : $Z^{(*)} \to \nu\bar{\nu}$

3/1/2023 practical Dirac-Majorana Confusion Theorem 32



nu nubar

In general

(useful if p_1 and/or p_2 are known)

However, after integra...

(required if momenta p_1 and p_2 are unobservable)

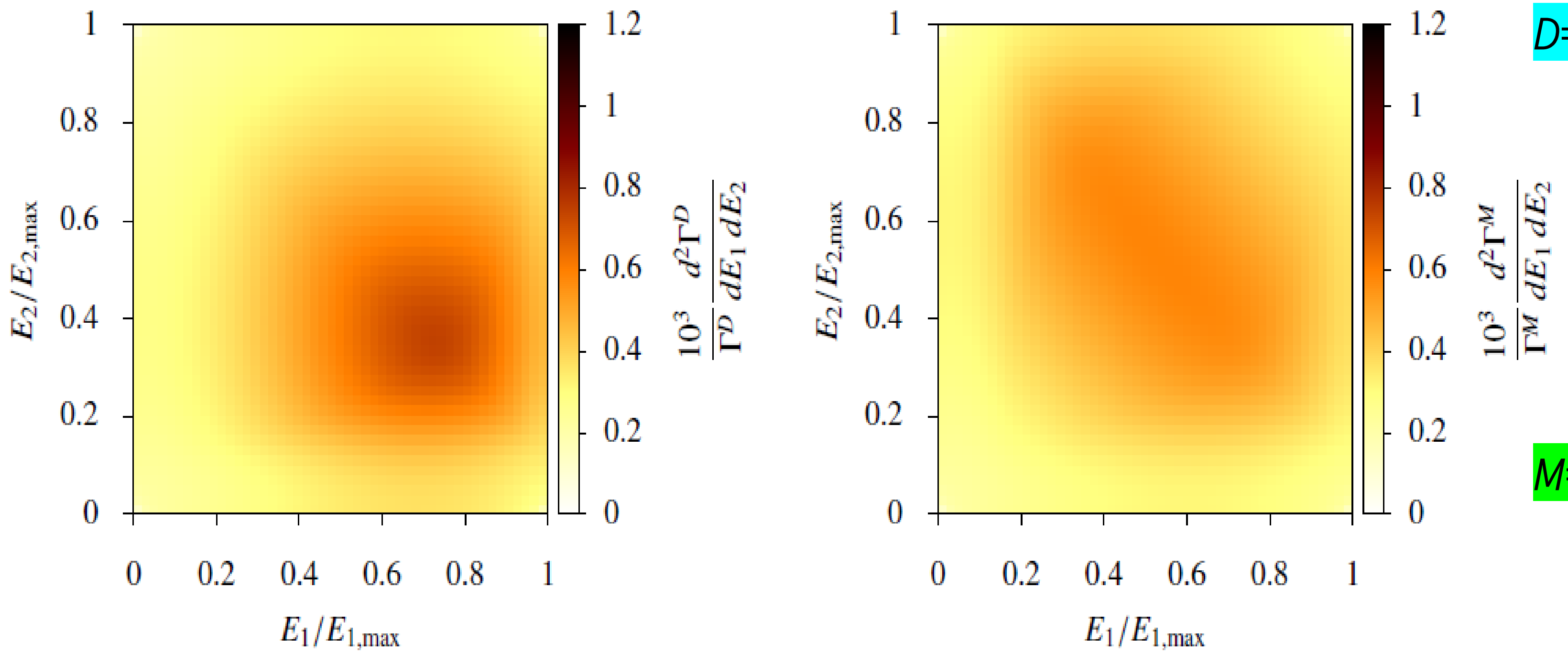
Direct term

Different distribution, but the same total rate (DMCT)

$$\mathscr{M}^D = \mathscr{M}(p_1, p_2), \qquad \mathscr{M}^M = \frac{1}{\sqrt{2}}\big(\mathscr{M}(p_1, p_2) - \mathscr{M}(p_2, p_1)\big).$$

Dirac case

Majorana case

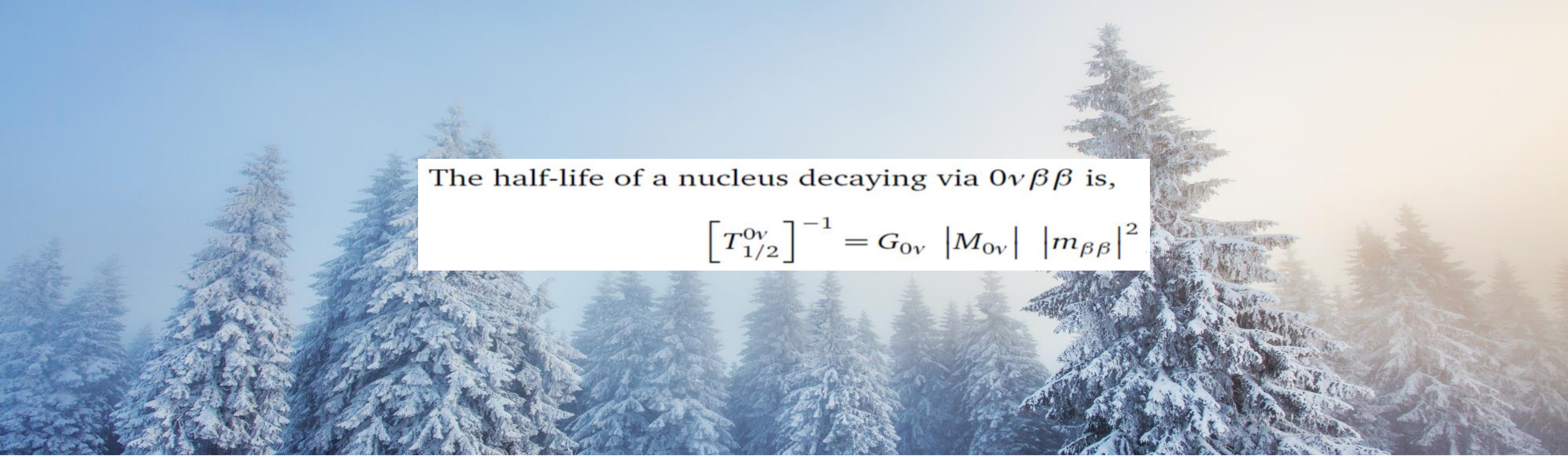


D=in general, no reason to be symmetric

M=always, must be symmetric

$$\underbrace{|\mathscr{M}(p_1, p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathscr{M}(p_2, p_1)|^2}_{\text{Exchange term}}.$$

$$\iint \left(\left|\mathscr{M}^D\right|^2 - \left|\mathscr{M}^M\right|^2\right) \mathrm{d}^4 p_1\, \mathrm{d}^4 p_2 \quad \propto m_\nu^2.$$



The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\ \left|M_{0\nu}\right|\ \left|m_{\beta\beta}\right|^2$$

How to overcome pDMCT ?

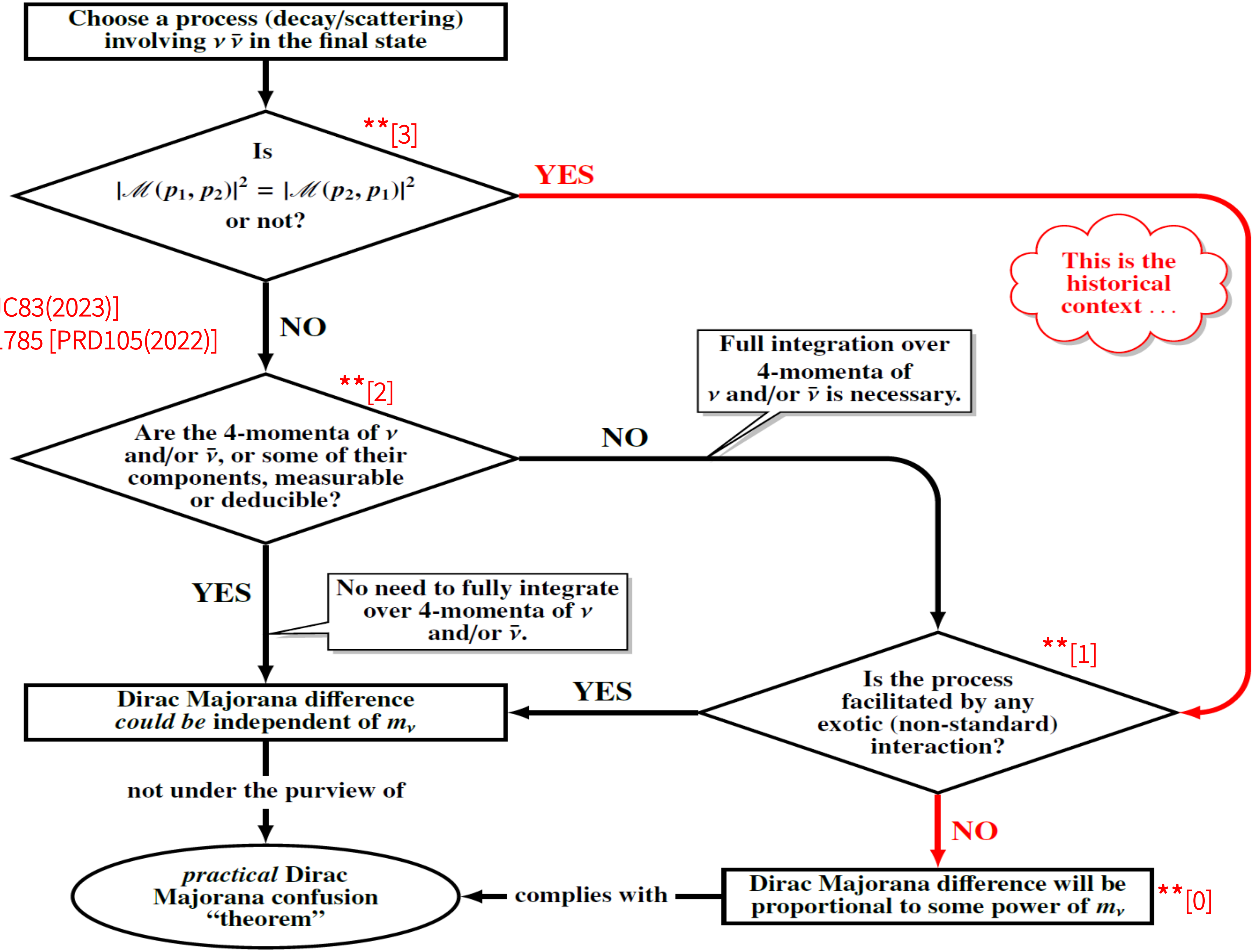
New physics effects in neutrino interaction (beyond SM)

[C. S.Kim, J.Rosiek, D.Sahoo, arXiv:2209.10110 [EPJC83 (2023)]; C. S.Kim, D.Sahoo, K.Vishnudath, arXiv:2405.17341 [EPJC84 (2024)]]

$B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, and deduction of neutrino energy-momenta (within SM)

[C.S. Kim, M. Murthy, D. Sahoo, arXiv:2106.11785 [PRD105(2022)]; C.S. Kim, arXiv:2307.05654 [EPJC83(2023)]]

Issues in pDMCT (Analytic Continuity & F-D Statistics) [C.S. Kim, arXiv:2307.05654 [EPJC83(2023)]



[0] B.Kayser,PRD26(1982)
[1] S.P.Rosen,PRL48(1982); CSK etal. 2209.10110 [EPJC83(2023)]
[2] T.Chabra,PR.Babu,PRD46(1992); CSK etal. 2106.11785 [PRD105(2022)]
[3] CS Kim, 2307.05654 (2023) [EPJC83(2023)]

Extra 1 [C. S.Kim, J.Rosiek, D.Sahoo, arXiv:2209.10110 [EPJC83 (2023)];
C. S.Kim, D.Sahoo, K.Vishnudath, arXiv:2405.17341 [EPJC84 (2024)]]

New Physics Effects to distinguish Majorana from Dirac

General Comments on New Physics Scenario [S.P.Rosen,PRL48(1982): W.Rodejohann etal,JHEP05(2017)]

Detailed study on $Z \to \nu_\ell \bar{\nu}_\ell$

Detailed study of $B \to K\nu\bar{\nu}, K \to \pi\nu\bar{\nu}$

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (1)

Most general amplitude for Dirac neutrino

$$\mathcal{M}^D = \mathcal{M}(p_1, p_2) = -\frac{i\, g_Z}{2} \epsilon_\alpha \left[\bar{u}(p_1)\, \gamma^\alpha \left(C_V^\ell - C_A^\ell \gamma^5\right) v(p_2)\right],$$

$$\boxed{C_{V,A}^\ell = \frac{1}{2} + \varepsilon_{V,A}^\ell,} \qquad \varepsilon_{V,A}^\ell = 0. \text{ (SM)}$$

Then,

$$\mathcal{M}^M = \frac{1}{\sqrt{2}}\left(\mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1)\right) = \frac{i\, g_Z\, C_A^\ell}{\sqrt{2}} \epsilon_\alpha \left[\bar{u}(p_1)\, \gamma^\alpha \gamma^5\, v(p_2)\right].$$

$$\left|\mathcal{M}^D\right|^2 = \frac{g_Z^2}{3}\left(\left((C_V^\ell)^2 + (C_A^\ell)^2\right)\left(m_Z^2 - m_\nu^2\right) + 3\left((C_V^\ell)^2 - (C_A^\ell)^2\right) m_\nu^2\right),$$

$$\left|\mathcal{M}^M\right|^2 = \frac{2\, g_Z^2\, (C_A^\ell)^2}{3}\left(m_Z^2 - 4\, m_\nu^2\right),$$

such that

$$\left|\mathcal{M}^D\right|^2 - \left|\mathcal{M}^M\right|^2 = \frac{g_Z^2}{3}\left(\left((C_V^\ell)^2 - (C_A^\ell)^2\right)\left(m_Z^2 + 2\, m_\nu^2\right) + 6\,(C_A^\ell)^2\, m_\nu^2\right)$$

$$= \begin{cases} \dfrac{g_Z^2}{2} m_\nu^2, & \text{(for SM alone)} \\[2ex] \dfrac{g_Z^2}{3}\left(\varepsilon_V^\ell - \varepsilon_A^\ell\right) m_Z^2, & \text{(with NP but neglecting } m_\nu) \end{cases}$$

[DMCT at amplitude^2]

[DMCT x with NP]

Detailed study of $Z \to \nu_\ell \bar{\nu}_\ell$ (2)

Therefore, neglecting neutrino mass,

$$\Gamma^D(Z \to \nu_\ell \bar{\nu}_\ell) = \Gamma_Z^0 \left(1 + 2\,\varepsilon_V^\ell + 2\,\varepsilon_A^\ell\right),$$

$$\Gamma^M(Z \to \nu_\ell \bar{\nu}_\ell) = \Gamma_Z^0 \left(1 + 4\,\varepsilon_A^\ell\right),$$

$$\Gamma_Z^0 = \frac{G_F\, m_Z^3}{12\sqrt{2}\,\pi},$$

Then,

$$\Gamma_{Z,\mathrm{inv}} = \begin{cases} \Gamma_Z^0 \left(3 + 2 \sum_{\ell=e,\mu,\tau} \left(\varepsilon_V^\ell + \varepsilon_A^\ell\right)\right), & \text{(for Dirac neutrinos)} \\ \Gamma_Z^0 \left(3 + 4 \sum_{\ell=e,\mu,\tau} \varepsilon_A^\ell\right). & \text{(for Majorana neutrinos)} \end{cases}$$

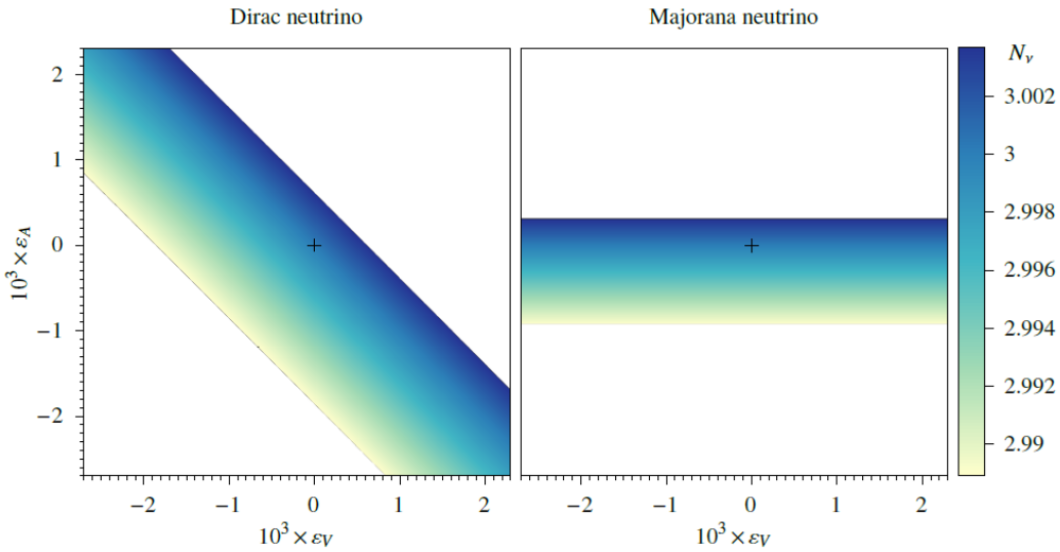
$$N_\nu = \Gamma_{Z,\mathrm{inv}} / \Gamma_Z^0 = 2.9963 \pm 0.0074.$$

[pdg(2021), PLB803(2020)]

$$\sum_{\ell=e,\mu,\tau} \left(\varepsilon_V^\ell + \varepsilon_A^\ell\right) = -0.0018 \pm 0.0037, \quad \text{(for Dirac neutrinos)}$$

$$\sum_{\ell=e,\mu,\tau} \varepsilon_A^\ell = -0.0009 \pm 0.0018, \quad \text{(for Majorana neutrinos)}$$

$$\varepsilon_{V,A}^e = \varepsilon_{V,A}^\mu = \varepsilon_{V,A}^\tau \equiv \varepsilon_{V,A}$$



Extra 2 [C.S. Kim, M. Murthy, D. Sahoo, arXiv:2106.11785 [PRD105(2022)];
C.S. Kim, arXiv:2307.05654 [EPJC83(2023)]]

BACK-TO-BACK muons (ie. B2B $\nu - \bar{\nu}$)

in $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2)$, decay.

Doubly weak charged current process in SM

$$\underbrace{|\mathcal{M}(p_1,p_2)|^2}_{\text{Direct term}} \neq \underbrace{|\mathcal{M}(p_2,p_1)|^2}_{\text{Exchange term}} .$$

B^0 (B_s)→ mu mu(Z^* → nu nu)
not good, too small BR (FCNC)

- Exception to pDMCT within the SM

Back-to-back muons, (easily measurable exception to pDMCT)

[CSK,MM,DS, 2106.11785, PRD105,113005(2022)]

[CS Kim, 2307.05654, EPJC83(2023)]

Allowed within SM

Majorana neutrinos

Quantum statistics

Final state

$\nu\ \nu$ $\overline{\nu}\ \overline{\nu}$ $\nu\ \overline{\nu}$

Anti-symmetrization for both Dirac and Majorana cases

Can not be used to distinguish between Dirac and Majorana cases

Anti-symmetrization for only Majorana case

Can be used to distinguish between Dirac and Majorana cases

Minimal final state: $\ell^- \ell^+ \nu_\ell \overline{\nu}_\ell$

Preference: $\ell = \mu$. $\ell = e$ has shorter track and $\ell = \tau$ has low reconstruction efficiency.

Decay mode: $B^0 \to \mu^- \mu^+ \nu_\mu \overline{\nu}_\mu$

Experiment: Belle II.

$\overline{B}$ is hadronically reconstructed.

(1 to 4 body process)

Many observables to deduce E(nu) !!

B^0 (B_s)→ mu mu(Z^* → nu nu) not good, too small BR (FCNC)

Back-to-back muons, (easily measurable exception to pDMCT)

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

In the rest frame
IF muon- and m
➔ nu and nu-b

t opposite direction

$E_1 = E_2$

ble or measurable ,

and $\mu_- - \mu_+$)

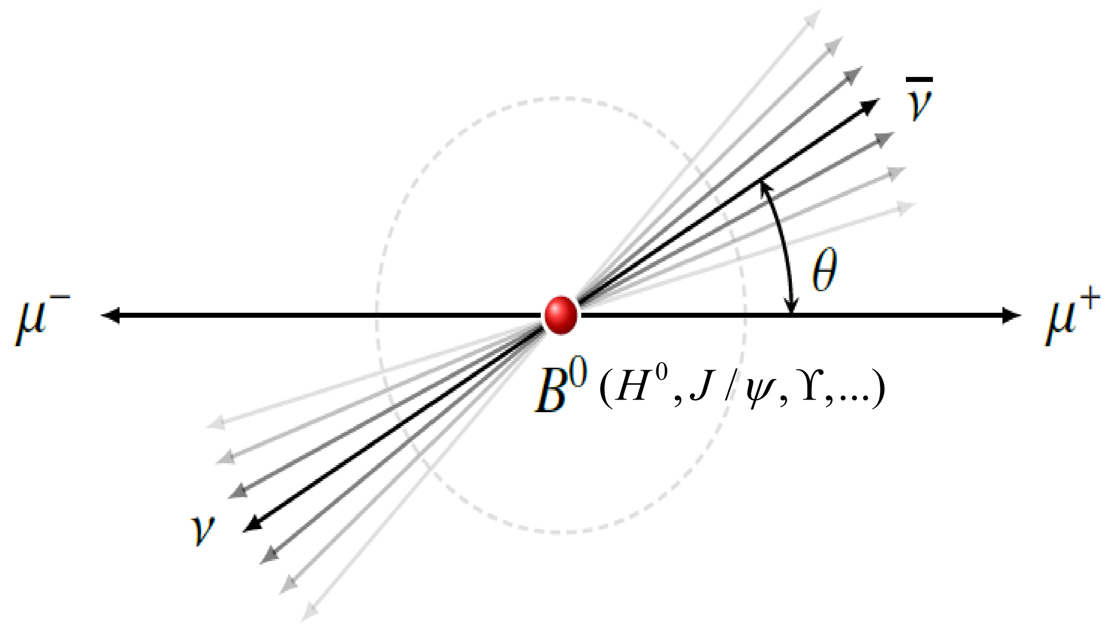
$m^2_{\mu\mu}$

$Y_m = \sqrt{(}$

Y_n

$$\frac{d\Gamma}{dE_\nu^2\, d\sin\theta}$$

Back-to-back geometry: $\vec{p}_1 = -\vec{p}_2$



See Sec.IV of Phys. Rev. D **105**, no.11, 113006 (2022).

Need not
only unmeasurable angle (θ) integrate out ➔ Overcoming DMCT constraint

Back-to-back muons, (easily measurable exception to DMCT)

$$B^0(p_B) \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$$

In the rest frame of parent B meson,
IF muon- and muon+ are back-to back, ie. flying with 3 momenta of equal magnitude but opposite direction
➔ nu and nu-bar also back-to-back

$$E_1 = E_2 = E_\nu = m_B/2 - E_\mu$$

$$m_{\nu\nu}^2 = 4E_\nu^2$$

$$m_{\mu\mu}^2 = (m_B - 2E_\nu)^2$$

$$Y_m = \sqrt{(m_B/2 - E_\nu)^2 - m_\mu^2}$$

$$Y_n = \sqrt{E_\nu^2 - m_\nu^2}$$

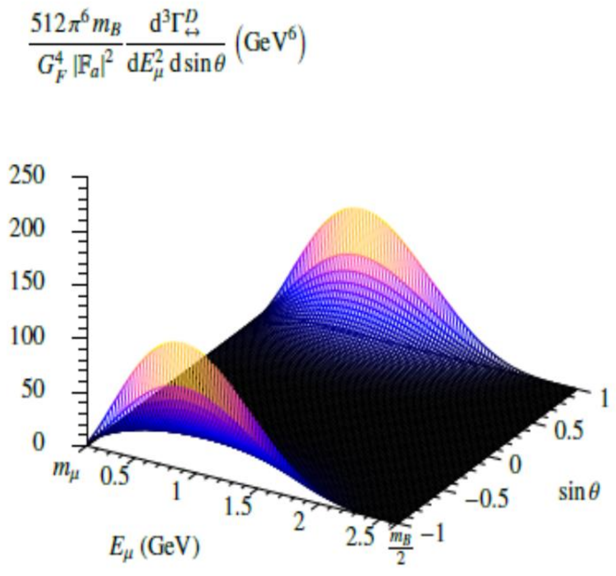
➔ All kinematic variables are calculable or measurable ,

Only the angle (between $\nu - \bar{\nu}$ and $\mu_- - \mu_+$) UNKNOWN

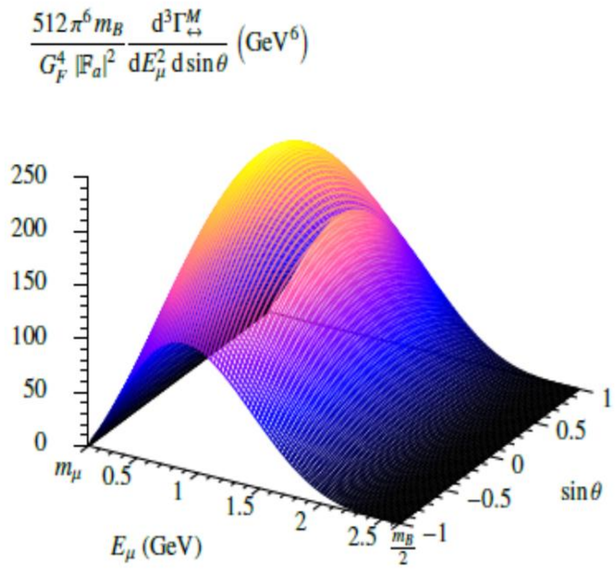
$$\boxed{\frac{d\Gamma}{dE_\mu^2\, d\sin\theta}} \to \boxed{\frac{d\Gamma}{dE_\nu^2\, d\sin\theta}}$$

Need not integrate out nu-nubar full phase space,
only unmeasurable angle (θ) integrate out ➔ Overcoming DMCT constraint

Back-to-back muons in $B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2),$



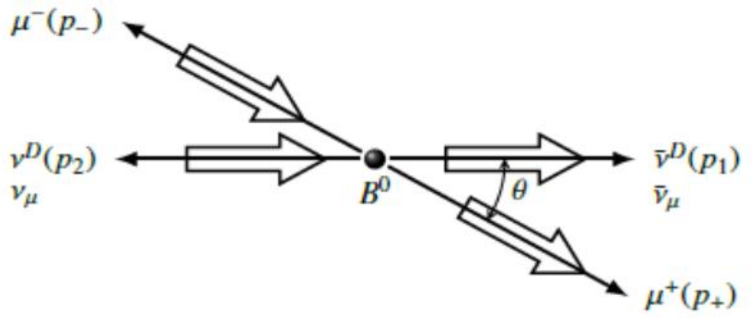
(a) Three dimensional view of the differential decay rate for Dirac case with an appropriate normalization as mentioned.



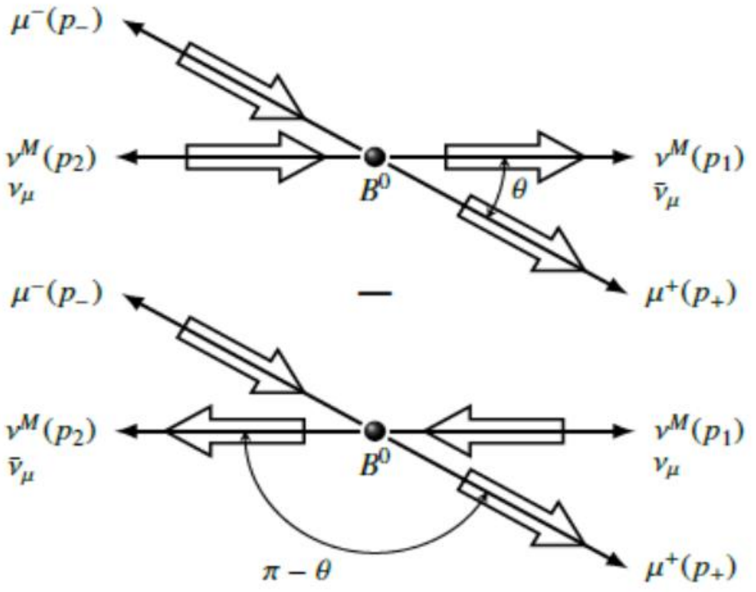
(b) Three dimensional view of the differential decay rate for Majorana case with an appropriate normalization as mentioned.

$$|\mathcal{M}^D_{\leftrightarrow}|^2 \propto \underbrace{\left(1-\cos\theta\right)^2}_{\text{Direct term}}.$$

$$|\mathcal{M}^M_{\leftrightarrow}|^2 \propto \frac{1}{2}\Big[\underbrace{\left(1-\cos\theta\right)^2}_{\text{Direct term}}+\underbrace{\left(1-\cos(\pi-\theta)\right)^2}_{\text{Exchange term}}-\underbrace{O\left(m_\nu^2\right)}_{\text{Interference term}}\Big] \simeq 1+\cos^2\theta.$$



(a) Helicity configuration involving Dirac neutrinos, $\nu_\mu \equiv \nu^D$, $\bar{\nu}_\mu \equiv \bar{\nu}^D$.



(b) Helicity configuration involving Majorana neutrinos, $\nu_\mu = \bar{\nu}_\mu \equiv \nu^M$.

Extra 3 [C.S. Kim, arXiv:2307.05654 [EPJC83(2023)]]

- Issues in pDMCT (Analytic Continuity & F-D statistics)

1. Analytic Continuity

   Is there any one-to-one correspondence between Dirac and Majorana neutrinos in the massless limit, $m_\nu \to 0$?

2. Anti-Symmetrization of Amplitude

   Should the amplitude be anti-symmetrized for pair of Majorana neutrinos of the same flavor ?

Analytic Continuity when m → 0 limit ? (1)

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

1) *In the context of a specific process and observable,*
there is indeed no one-to-one correspondence between the Dirac and Majorana neutrino scenarios.

E.g. For $Z \to \nu_\ell \bar{\nu}_\ell$

$$|\mathcal{M}^D|^2 - |\mathcal{M}^M|^2 = \frac{g_Z^2}{3}\left(\left((C_V^\ell)^2 - (C_A^\ell)^2\right)\left(m_Z^2 + 2\,m_\nu^2\right) + 6\,(C_A^\ell)^2\,m_\nu^2\right)$$

$$= \begin{cases} \dfrac{g_Z^2}{2} m_\nu^2, & \text{(for SM alone)} \\ \dfrac{g_Z^2}{3}\left(\varepsilon_V^\ell - \varepsilon_A^\ell\right) m_Z^2, & \text{(with NP but neglecting } m_\nu) \end{cases}$$

$$C_{V,A}^\ell = \frac{1}{2} + \varepsilon_{V,A}^\ell, \qquad \varepsilon_{V,A}^\ell = 0.\ \text{(SM)}$$

Analytic Continuity when m → 0 limit ?   (2)

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

2) *Now assuming Standard Model Lagrangian*, **due to the (V-A) structure of weak interaction,**
always **only** $\nu_L, \bar{\nu}_R$ **are produced.**
Assuming $mass(\nu) = 0$, **chirality of neutrinos is conserved and invariant.**
And $\nu_L, \bar{\nu}_R$ *can be* **distinguished by weak charged current interaction.**

➔ **massless** $\nu_L, \bar{\nu}_R$ are **Dirac neutrino** (No mass-less Majorana neutrino exists).

3) *Assuming* $mass(\nu) \neq 0$, **chirality of neutrinos is not conserved (still invariant).**

space-time evolution

$$i\gamma^\mu \partial_\mu \psi_R = m\,\psi_L,$$
$$i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R.$$

$$\psi_L \to \begin{pmatrix} \frac{m}{E}\psi_R \\ \psi_L \end{pmatrix}$$

$$\begin{Bmatrix} m \to 0 \\ E \neq 0 \end{Bmatrix} : \text{relativistic} \qquad \frac{m}{E} \to 0$$

$$\begin{Bmatrix} m \to 0 \\ E \to 0 \end{Bmatrix} : \text{non-relativistic} \qquad \frac{m}{E} \sim \text{finite}$$

(relevant parameter, not m, but m/E → 0)

Analytic Continuity when m → 0 limit ?   (3)

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

4) *Most importantly, **Quantum Statistics DEPENDS** on **Spin & Identicalness of neutrino**,*
***not DEPENDS** on **Mass of neutrino** !!*

E.g. For $$B^0(p_B) \to \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2),$$

if $\nu = \text{Dirac}$ $$\mathcal{M}^D = \mathcal{M}(p_1, p_2),$$

if $\nu = \text{Majorana}$ $$\mathcal{M}^M = \frac{1}{\sqrt{2}}\left(\mathcal{M}(p_1, p_2) - \mathcal{M}(p_2, p_1)\right).$$ (for $m_\nu = 0$ or $m_\nu \neq 0$)

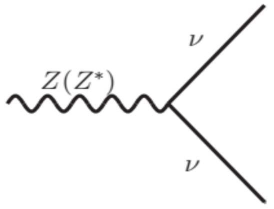
Therefore, even if $\Gamma_D = \Gamma_M(m_\nu \to 0)$, $d\Gamma_D \neq d\Gamma_M(m_\nu \to 0)$

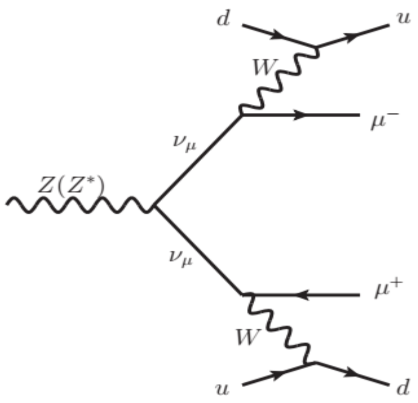
Anti-symmetrization of Amplitude : F-D Statistics

** Should the amplitude be anti-symmetrized for pair of Majorana neutrinos of the same flavor ??

1) Quantum statistics requires absolutely identical indistinguishable particles.

2) Quantum measurement effect is analogous to the well-known fact that **the interference pattern in a double slit experiment is destroyed if one could somehow know through which slit the photon has passed.**

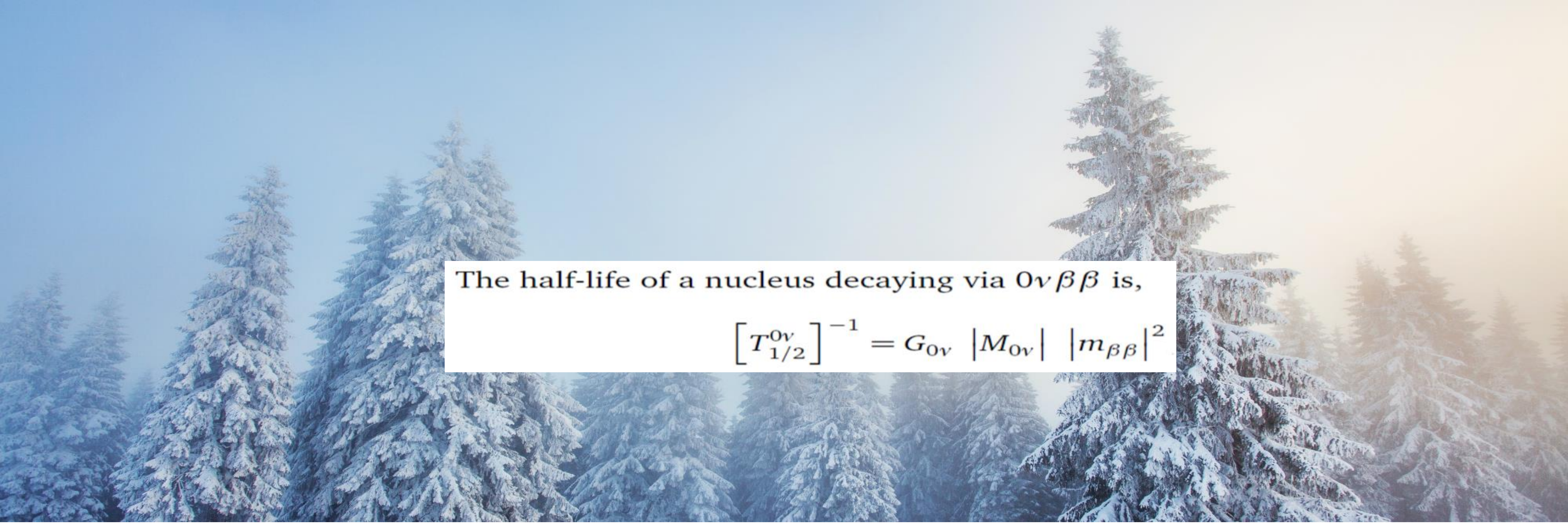




➔ requires a method deducing E (or p) of neutrino without destroying quantum effects.

Need Anti-symmetrization

No Anti-symmetrization once we know $\nu_1 \neq \nu_2$



The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\ \left|M_{0\nu}\right|\ \left|m_{\beta\beta}\right|^2$$

Alternatives to $0\nu\beta\beta$ ($\Delta L = 2$ process)

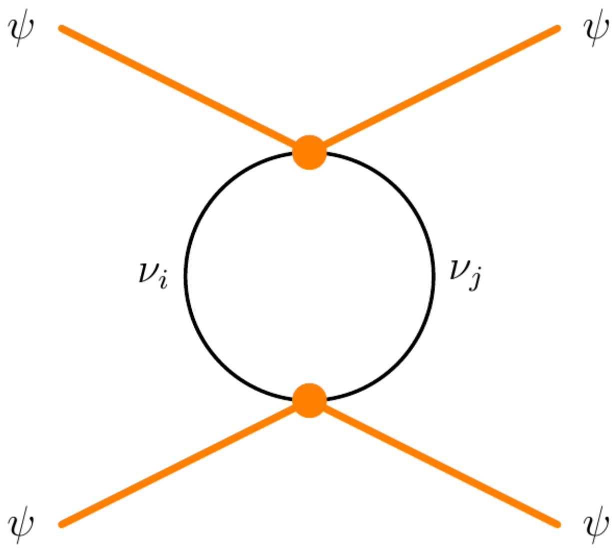
Neutrino Casimir Force
EDM & MDM of neutrinos
Fermi-Dirac Statistics for Fermion (nu) – Quantum Statistics
Quantum Statistics + $(2\nu\beta\beta)$

Alternative to 0nuBB (1) – Neutrino Casimir force

Principle: Exchange of pair of neutrinos can give rise to long-range quantum force (aka neutrino Casimir force or the neutrino exchange force) between macroscopic objects, and the effective potential can differentiate Dirac and Majorana neutrinos.

G Feinberg, J Sucher, PRD166(1968)
Phys. Rev. D 101, no.11, 116006 (2020); JHEP 09, 122 (2020)
arXiv:2209.07082 [hep-ph]

ν-Casimir force



Issue: The potential (and hence the force) is proportional to product of the tiny neutrino masses in the loop.

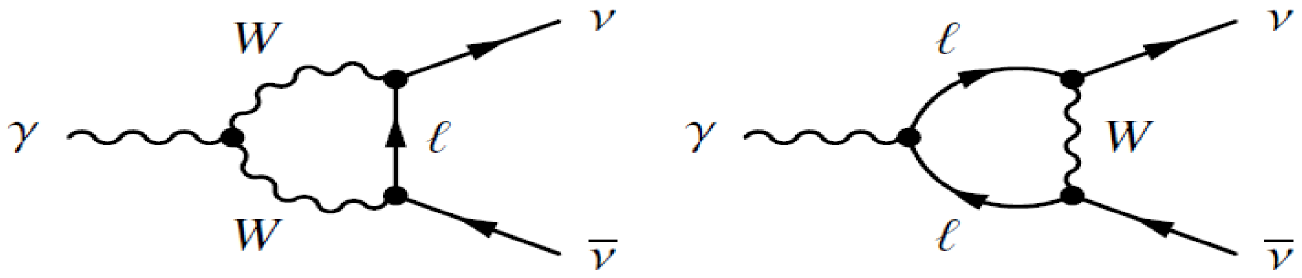
** Thermal fluctuation, van der Waals force

** Very weak force. For $r > 1$ nm, Gravitational force between two protons is bigger than this force.

Status: Experimental study is still awaited.

Alternative to 0nuBB (2) – e.d.m. & m.d.m. of Neutrino

- Neutrinos: electrically neutral ($Q_\nu = 0$).
  But might carry non-zero electric and magnetic dipole moments.
- Neutrino has no interaction with photon at tree-level.
  At one loop-level involving charged lepton and W boson in loop such interaction is allowed.



- CPT invariance prohibits Majorana neutrinos to have any electric and magnetic dipole moment. Loop effects also confirm this, following quantum statistics in $\gamma^* \to \nu\overline{\nu}$.

$$\text{edm} = 0 = \text{mdm}, \quad \text{(Majorana)}$$
$$\text{edm} \neq 0 \neq \text{mdm}. \quad \text{(Dirac)}$$

- Such striking differences in electromagnetic properties of Dirac and Majorana neutrinos vanishes as $m_\nu \to 0 \implies$ DMCT.
  [B. Kayser, Phys. Rev. D **26**, 1662 (1982).]

Alternative to 0nuBB (3) – Quantum Statistics

[CSK,MM,DS, 2106.11785, PRD105,113005(2022)]

[CS Kim, 2307.05654, EPJC83(2023)]

Majorana neutrinos

Quantum statistics

Final state

$\nu\ \nu$ $\quad \overline{\nu}\ \overline{\nu}$ $\quad \nu\ \overline{\nu}$

Anti-symmetrization for both Dirac and Majorana cases

Anti-symmetrization for only Majorana case

Can not be used to distinguish between Dirac and Majorana cases

Can be used to distinguish between Dirac and Majorana cases

Allowed within SM

Minimal final state: $\ell^-\ell^+\nu_\ell\overline{\nu}_\ell$

Preference: $\ell=\mu$. $\ell=e$ has shorter track and $\ell=\tau$ has low reconstruction efficiency.

Decay mode: $B^0\to\mu^-\mu^+\nu_\mu\overline{\nu}_\mu$

Experiment: Belle II.

$\overline{B}$ is hadronically reconstructed.

(1 to 4 body process)

Many observables to deduce E(nu) !!

B^0 (B_s)→ mu mu(Z^* → nu nu)

not good, too small BR (FCNC)

Alternative to 0nuBB (4) – Quantum Statistics + $(2\nu\beta\beta)$

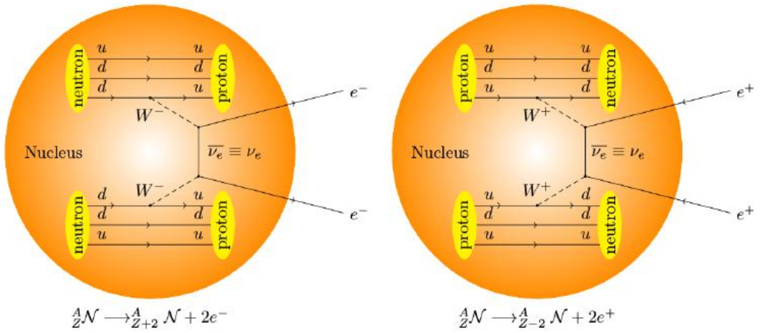
$0\nu\beta\beta$ is only for the evidence of
"Lepton Number Violation",

➔ NOT unequivocal evidence of
active (sub-eV) neutrino being
Majorana neutrino .

eg. heavy sterile neutrinos can give the same effects

HOWEVER, $2\nu\beta^+\beta^-$ ($\nu\bar{\nu}$ not $\nu\nu, \bar{\nu}\bar{\nu}$)
can be w/ Quantum Statistics !!

❖ Process:



$${}^A_Z N \rightarrow {}_{Z+2}^{\ \ A} N + 2e^-$$

$${}^A_Z N \rightarrow {}_{Z+2}^{\ \ A} N + 2e^- + (2\bar{\nu})$$

$${}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$ ✖

$$e^- + {}^A_Z N \rightarrow {}^A_Z N + e^- + (\bar{\nu} + \nu)$$ –

$$e^+ + {}^A_Z N \rightarrow {}^A_Z N + e^+ + (\bar{\nu} + \nu)$$ –

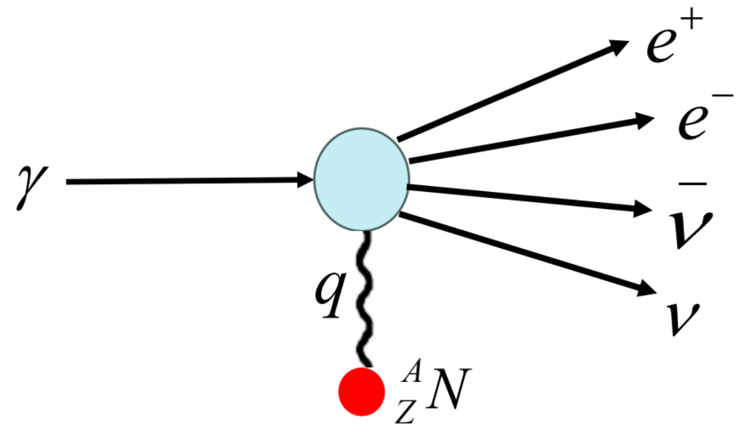
$$\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$ ○

Alternative to 0nuBB (5) – Quantum Statistics + $(2\nu\beta\beta)$

γ = (squeezed) Gamma-ray Laser Photon
[Compton back-scattered gamma ray]

HOWEVER, $2\nu\beta^+\beta^-$ ($\nu\bar{\nu}$ not $\nu\nu, \bar{\nu}\bar{\nu}$)
can be w/ Quantum Statistics !!

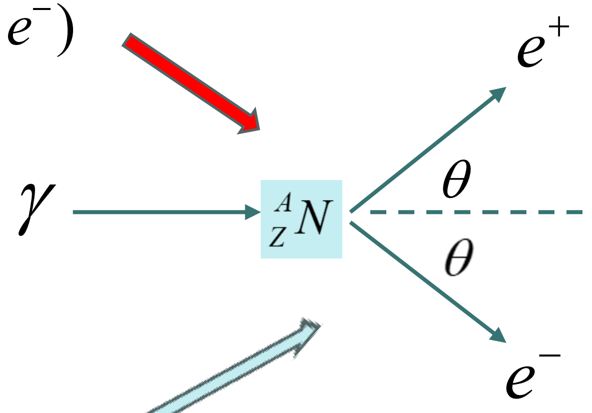
$$\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$



$$E(\gamma) \geq M({}^A_Z N^*) - M({}^A_Z N)$$

$$\gamma + {}^A_Z N \to {}^A_Z N^* \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

$$\to {}^A_Z N + \gamma^* (\to e^+ + e^-)$$



$$1.02\text{MeV} < E(\gamma) < M({}^A_Z N^*) - M({}^A_Z N)$$

$$\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

$$\to {}^A_Z N + \gamma^* (\to e^+ + e^-)$$

$E_\gamma \approx 5 - 10\text{MeV}$

$\lambda(\gamma) \sim 100\text{fm} \quad \lambda(\text{p,n}) \sim 1\text{fm}$

$[\gamma + N \to N][\gamma^* \to ee\nu\nu]$

$[\gamma + N \to N][N \to N + ee\nu\nu]$

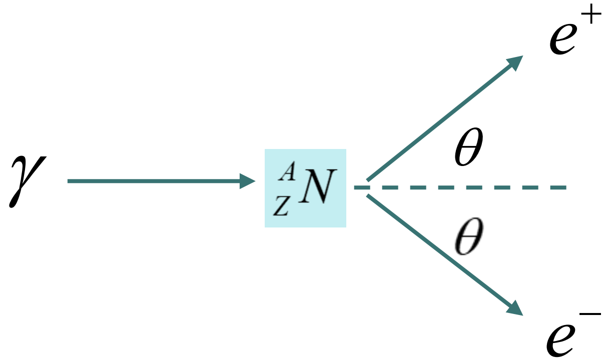
E.g. Electron-Positron Production

In free space (vacuum) : $\gamma \rightarrow e^{+} + e^{-}$

With matter (nucleus) : $\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{+} + e^{-}$

$$1.02\mathrm{MeV} < E(\gamma) < M({}^{A}_{Z}N^{*}) - M({}^{A}_{Z}N)$$

$$\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + \gamma^{*}(\rightarrow e^{+} + e^{-})$$

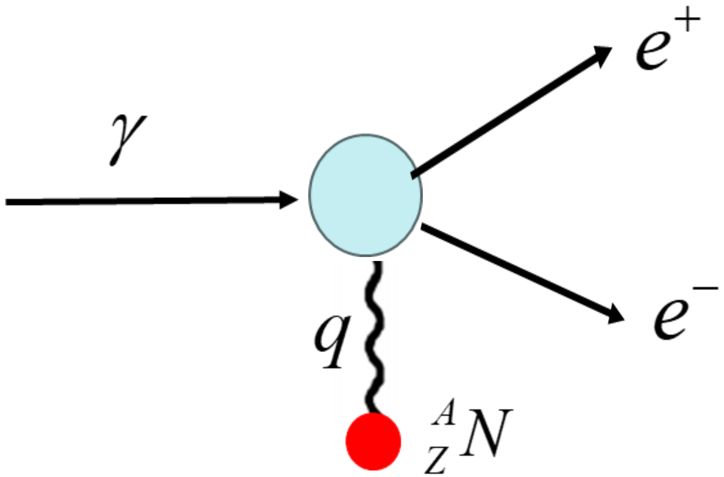


$$h\nu = 2m_{e} + E_{e^{+}} + E_{e^{-}}$$

$$p_{\gamma} = p_{e^{+}} + p_{e^{-}}$$

$$\vec{p}_{\gamma} = \vec{p}_{e^{+}} + \vec{p}_{e^{-}}$$

$$p^{2}_{\gamma^{*}} \approx q^{2}_{\min} \neq 0$$



$$\Gamma(\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{+} + e^{-}) \approx \mathrm{Prob.}(\gamma N) \times \Gamma(\gamma^{*}_{(p^{2}_{\gamma} \sim q^{2}_{\min})} \rightarrow e^{-} + e^{+})$$

C.S. Kim and Meng Ru Wu

BACK-TO-BACK ν and $\bar{\nu}$ in $2\nu\beta^+\beta^-$

$$\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

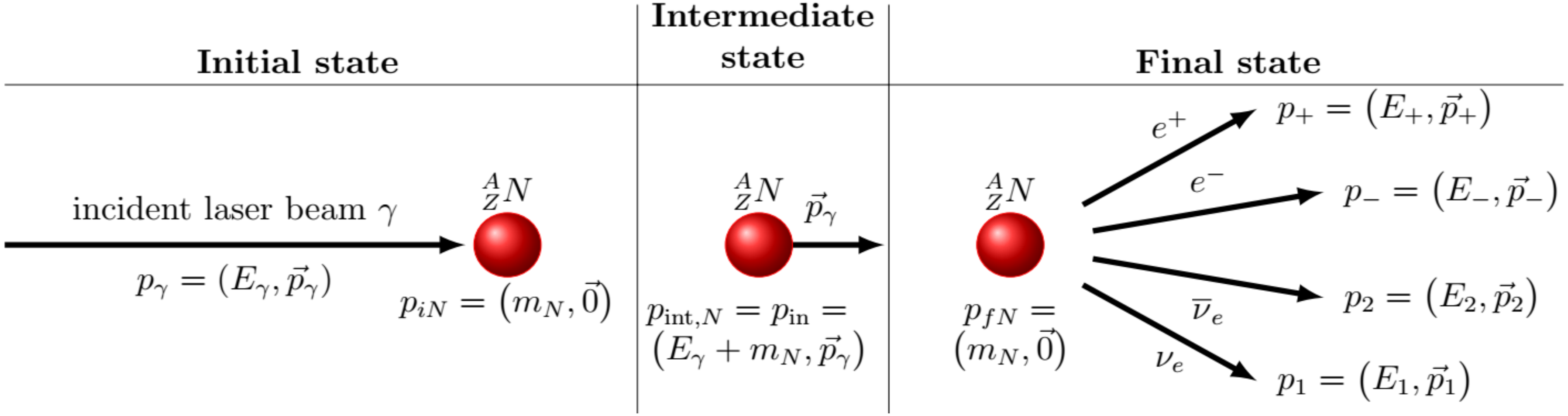
- experimental observable exception to pDMCT ;
- back-to-back $\nu - \bar{\nu}$

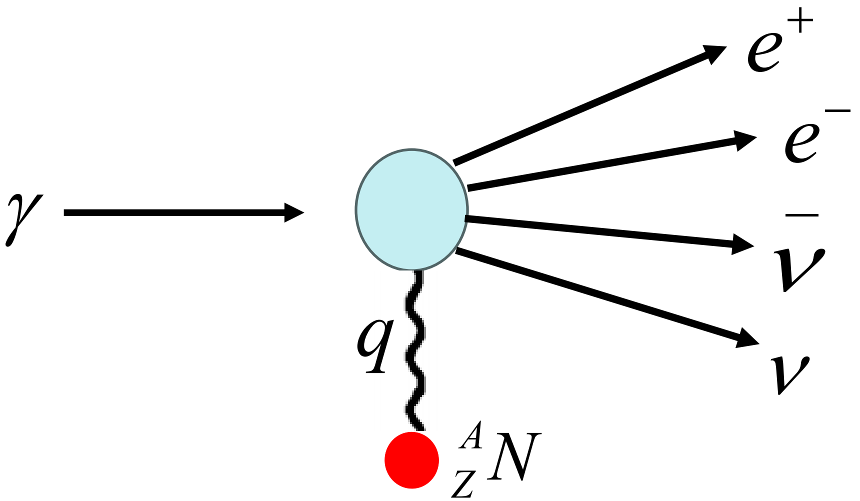
Kinematic study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$
Back-to-back kinematics in $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$
Detailed study of $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$
Background study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$

Kinematic study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (1)

$$\gamma + {}^A_Z N \to {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

Lab - frame





q^2 could be (-) or (+) depending on S- or T- process,

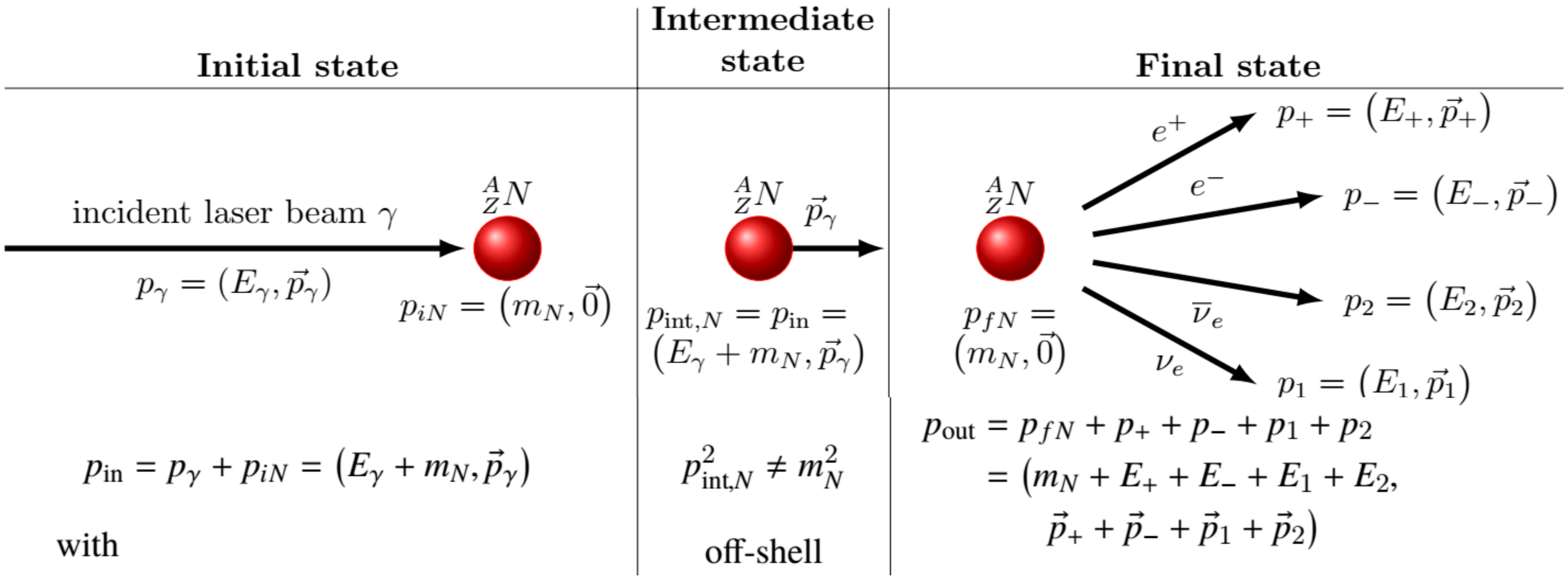
Amp ~ $1/q^2$, so maximal contribution at $q_{\min} \approx 1.02\mathrm{MeV}$

$p_\gamma^2 = 0$ (in free space) ➔ $(p_+ + p_- + p_1 + p_2)^2 \sim q_{\min}^2$

Kinematic study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (2)

$$\gamma + {}^A_Z N \to {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

Lab - frame



$$\begin{aligned} & p_{\mathrm{in}} = p_{\mathrm{out}} \\ \Longrightarrow\ & p_\gamma + p_{iN} = p_{fN} + p_+ + p_- + p_1 + p_2 \\ \Longrightarrow\ & \boxed{p_\gamma = p_+ + p_- + p_1 + p_2}\,. \end{aligned}$$

$$p^2_\gamma \sim q^2_{\min}$$

$$\boxed{\vec{p}_\gamma = \vec{p}_+ + \vec{p}_- + \vec{p}_1 + \vec{p}_2 \neq \vec{0}}\,.$$

Kinematic study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (3)

Lab - frame $\boxed{p_\gamma = p_+ + p_- + p_1 + p_2}$ $\boxed{\vec{p}_\gamma = \vec{p}_+ + \vec{p}_- + \vec{p}_1 + \vec{p}_2 \neq \vec{0}}$

Any Boosted - frame $p'_x = \Lambda\, p_x, \quad \text{for } x \in \{\gamma, iN, fN, +, -, 1, 2\}.$

back-to-back neutrino frame $\boxed{\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}}$

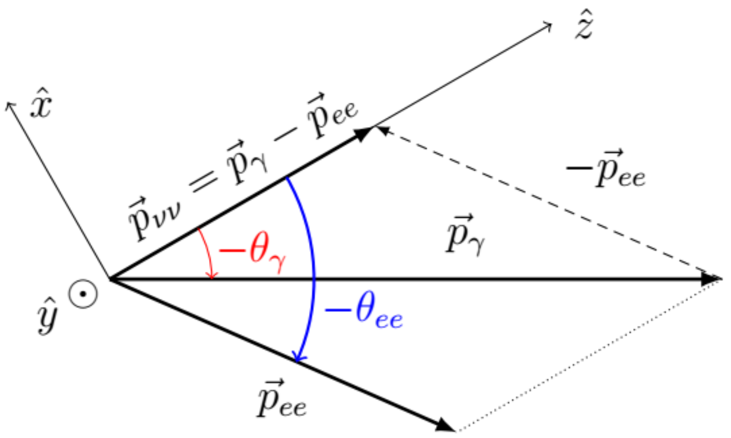
Realizing back-to-back neutrino condition

$$\boxed{\vec{p}_{\nu\nu} = \vec{0}} \qquad \vec{\beta}_{\nu\nu} = \frac{\vec{p}_{\nu\nu}}{E_{\nu\nu}}. \qquad \gamma_{\nu\nu} = \frac{1}{\sqrt{1 - |\vec{\beta}_{\nu\nu}|^2}} = \frac{E_{\nu\nu}}{m_{\nu\nu}}.$$

$$\begin{aligned}
p_{ee} &\equiv (E_{ee}, \vec{p}_{ee}) = p_+ + p_- = (E_+ + E_-, \vec{p}_+ + \vec{p}_-)\,, \\
p_{\nu\nu} &\equiv (E_{\nu\nu}, \vec{p}_{\nu\nu}) = p_1 + p_2 = (E_1 + E_2, \vec{p}_1 + \vec{p}_2)\,, \\
m_{ee}^2 &\equiv p_{ee}^2 = E_{ee}^2 - |\vec{p}_{ee}|^2\,, \\
m_{\nu\nu}^2 &\equiv p_{\nu\nu}^2 = E_{\nu\nu}^2 - |\vec{p}_{\nu\nu}|^2\,.
\end{aligned}$$

$$\begin{aligned}
p'_{\nu\nu} = \Lambda\, p_{\nu\nu} &= \begin{pmatrix} \gamma_{\nu\nu} & 0 & 0 & -\gamma_{\nu\nu}\beta_{\nu\nu} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_{\nu\nu}\beta_{\nu\nu} & 0 & 0 & \gamma_{\nu\nu} \end{pmatrix} \begin{pmatrix} E_{\nu\nu} \\ 0 \\ 0 \\ |\vec{p}_{\nu\nu}| \end{pmatrix} \\
&= \begin{pmatrix} \frac{E_{\nu\nu}}{m_{\nu\nu}} & 0 & 0 & -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} & 0 & 0 & \frac{E_{\nu\nu}}{m_{\nu\nu}} \end{pmatrix} \begin{pmatrix} E_{\nu\nu} \\ 0 \\ 0 \\ |\vec{p}_{\nu\nu}| \end{pmatrix} = \begin{pmatrix} m_{\nu\nu} \\ 0 \\ 0 \\ 0 \end{pmatrix}
\end{aligned}$$

Lab - frame



Kinematic study on $\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^- + e^+ + (\bar{\nu} + \nu)$ (4)

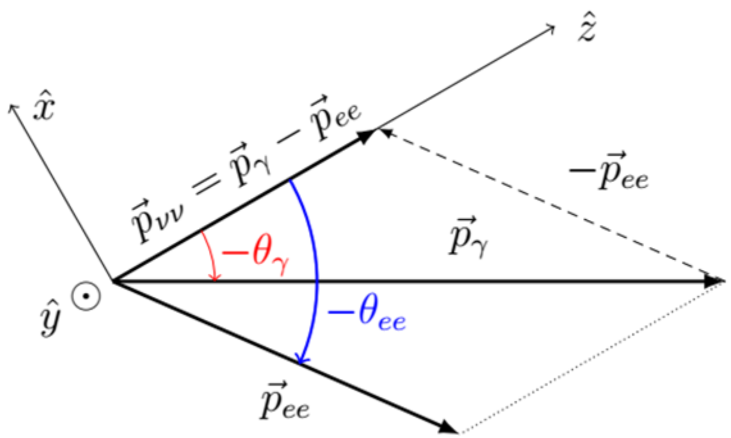
Lab - frame $\boxed{p_\gamma = p_+ + p_- + p_1 + p_2}$

Boosted (back-to-back neutrino) - frame

$$\boxed{\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}}$$

$$E_\nu' = m_{\nu\nu}/2 = \frac{\sqrt{(E_\gamma - E_+ - E_-)^2 - (\vec{p}_\gamma - \vec{p}_+ - \vec{p}_-)^2}}{2}$$

$$m_{\nu\nu} \in \left\{ 2\,m_\nu, \sqrt{\left(E_{ee} + |\vec{p}_{ee}| - 2\,E_\gamma\right)\left(E_{ee} - |\vec{p}_{ee}|\right)} \right\}.$$



$(E_\gamma, \vec{p}_\gamma), (E_+, \vec{p}_+), (E_-, \vec{p}_-), (E_{\nu\nu}, \vec{p}_{\nu\nu}), E_\nu'$ All observable, calculable !!

$$\frac{d\Gamma_{D/M}}{dE_\nu'}$$

Measurable, calculable !!

Lab - frame

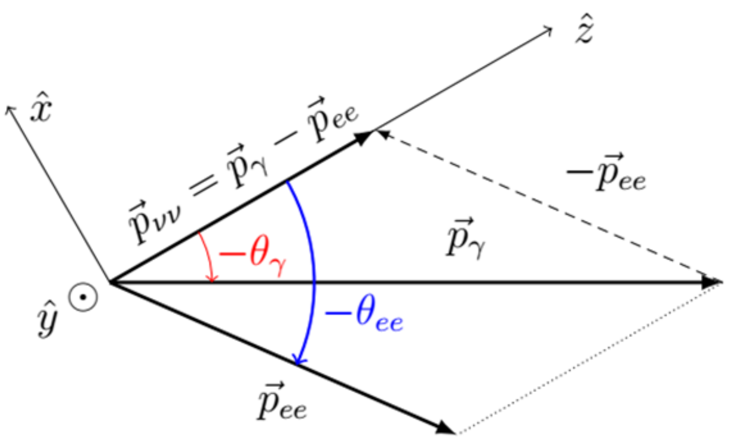
Kinematic study on $\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (5)

Lab - frame $\boxed{p_\gamma = p_+ + p_- + p_1 + p_2}$

Boosted (back-to-back neutrino) - frame $\boxed{\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}}$

$$p_x' = \Lambda\, p_x, \qquad \text{for } x \in \{\gamma, iN, fN, +, -, 1, 2\}.$$

$$\Lambda = \begin{pmatrix} \gamma_{\nu\nu} & 0 & 0 & -\gamma_{\nu\nu}|\vec{\beta}_{\nu\nu}| \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_{\nu\nu}|\vec{\beta}_{\nu\nu}| & 0 & 0 & \gamma_{\nu\nu} \end{pmatrix} = \begin{pmatrix} \frac{E_{\nu\nu}}{m_{\nu\nu}} & 0 & 0 & -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} & 0 & 0 & \frac{E_{\nu\nu}}{m_{\nu\nu}} \end{pmatrix}$$



Lab - frame

e.g.

$$p_{ee}' = \Lambda\, p_{ee} = \begin{pmatrix} \frac{E_{\nu\nu}}{m_{\nu\nu}} & 0 & 0 & -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{|\vec{p}_{\nu\nu}|}{m_{\nu\nu}} & 0 & 0 & \frac{E_{\nu\nu}}{m_{\nu\nu}} \end{pmatrix} \begin{pmatrix} E_{ee} \\ -|\vec{p}_{ee}| \sin\theta_{ee} \\ 0 \\ |\vec{p}_{ee}| \cos\theta_{ee} \end{pmatrix} = \begin{pmatrix} \frac{E_{\nu\nu}\, E_{ee}}{m_{\nu\nu}} - \frac{|\vec{p}_{\nu\nu}|\,|\vec{p}_{ee}|}{m_{\nu\nu}} \cos\theta_{ee} \\ -|\vec{p}_{ee}| \sin\theta_{ee} \\ 0 \\ -\frac{|\vec{p}_{\nu\nu}|\, E_{ee}}{m_{\nu\nu}} + \frac{E_{\nu\nu}\, |\vec{p}_{ee}|}{m_{\nu\nu}} \cos\theta_{ee} \end{pmatrix}$$

i.e. $$Q' = E_{e+}' + E_{e-}' = \frac{E_{\nu\nu}\, E_{ee}}{m_{\nu\nu}} - \frac{|\vec{p}_{\nu\nu}|\,|\vec{p}_{ee}|}{m_{\nu\nu}} \cos\theta_{ee}$$

All observable, calculable !!

Back-to-back neutrinos, (exception to DMCT) in (1)

$$\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{-} + e^{+} + (\bar{\nu} + \nu)$$

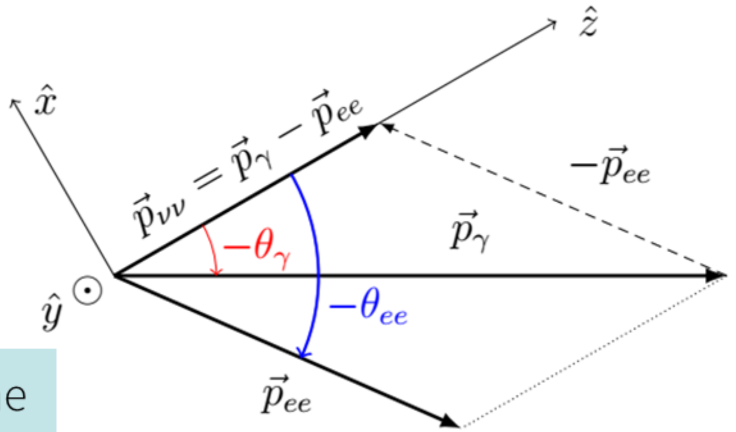
Boosted (back-to-back neutrino) - frame

$$\boxed{\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}}$$

<u>Back-to-back (B2B) Constraints</u> WITH Experimental Resolution, ΔE, $\Delta \vec{P}$.

$$\boxed{\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}}$$

$$E_{e^+,e^-} >> \Delta E \qquad E_{e^+} + E_{e^-} < E_\gamma - \Delta E \qquad \vec{p}_+' + \vec{p}_-' \approx \vec{p}_\gamma'$$



Lab - frame

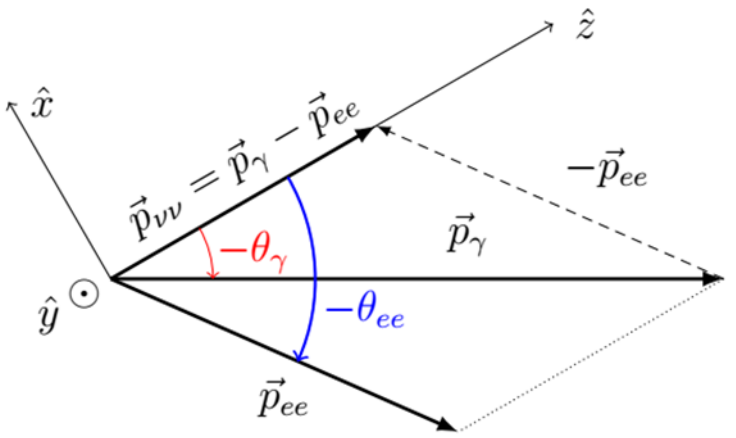
Measure $\dfrac{d\Gamma}{dE_\nu'}$ $\quad \dfrac{d\Gamma}{dE_{e^-+e^+}} = \dfrac{d\Gamma}{dQ}$ $\quad \dfrac{d\Gamma}{d\cos(\theta_{ee}')}$

Back-to-back neutrinos, (exception to DMCT) in (2)

$$\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$$

Lab - frame $p_\gamma = p_+ + p_- + p_1 + p_2$

Boosted (back-to-back neutrino) - frame $\vec{p}_1' + \vec{p}_2' = \vec{0}, \quad \text{only when} \quad \vec{p}_+' + \vec{p}_-' = \vec{p}_\gamma' \neq \vec{0}$



$$d\Gamma_{D/M} = |M_{D/M}|^2 \, d_4(PS : \gamma \rightarrow e^- + e^+ + (\bar{\nu} + \nu))$$

[Barger, Phillips, Collider Physics, Appendix B]

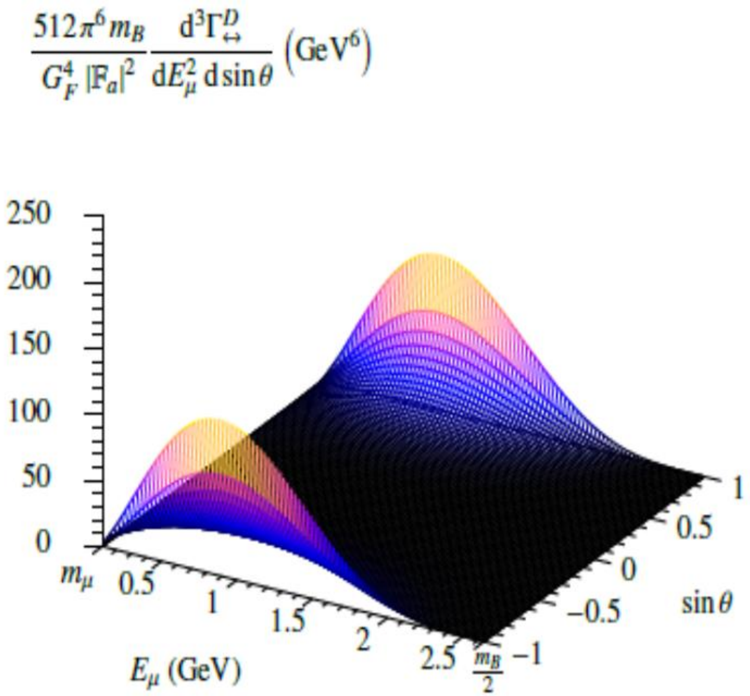
$$d_4(PS : \gamma \rightarrow e^- + e^+ + (\bar{\nu} + \nu)) = d_2(PS : \gamma \rightarrow XY) dm_X^2 dm_Y^2$$
$$\times d_2(PS : X \rightarrow e^- + e^+) d_2(PS : Y \rightarrow \bar{\nu} + \nu)$$

$$E_\nu' = m_{\nu\nu}/2 = \frac{\sqrt{(E_\gamma - E_+ - E_-)^2 - (\vec{p}_\gamma - \vec{p}_+ - \vec{p}_-)^2}}{2}$$

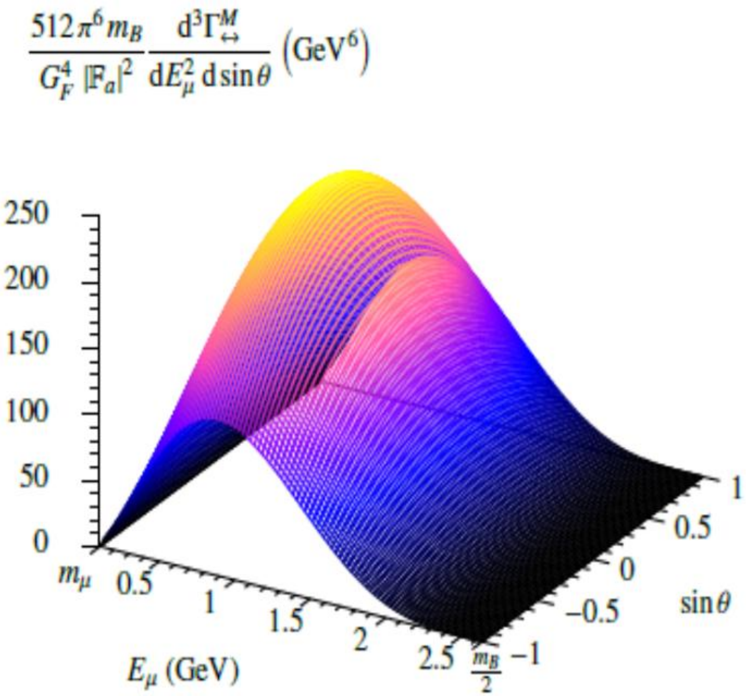
Calculate $\frac{d\Gamma_{D/M}}{dE_\nu'}$ $\frac{d\Gamma_{D/M}}{dE_{e^- + e^+}} = \frac{d\Gamma_{D/M}}{dQ}$ $\frac{d\Gamma_{D/M}}{d\cos(\theta_{ee}')}$ to compare D with M

Back-to-back neutrinos in $\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{-} + e^{+} + (\bar{\nu} + \nu)$ (3)

e.g.) B2B Muons in $B^0(p_B) \rightarrow \mu^-(p_-)\mu^+(p_+)\bar{\nu}_\mu(p_1)\nu_\mu(p_2),$



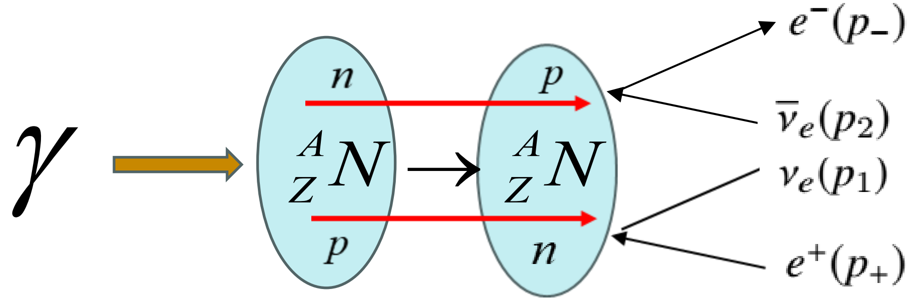
(a) Three dimensional view of the differential decay rate for Dirac case with an appropriate normalization as mentioned.



(b) Three dimensional view of the differential decay rate for Majorana case with an appropriate normalization as mentioned.

Detailed study on $\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (0) $|M^{D/M}|^2$

(A)



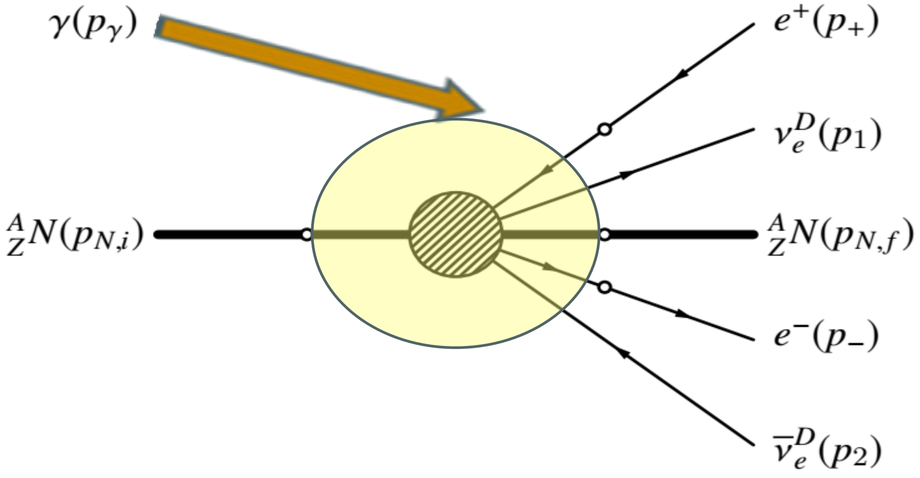
$[\gamma + N \rightarrow N][N \rightarrow N + ee\nu\nu]$ (A)

$E_\gamma \approx 5-10\text{MeV}$ $\lambda(\gamma) \sim 100\text{fm}$

$\lambda(\text{p,n}) \sim 1\text{fm}$

(B)

(C)

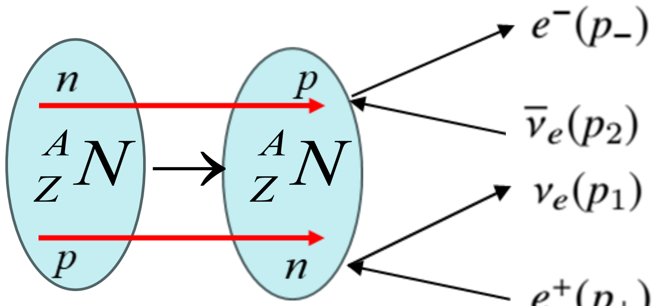


$[\gamma + N \rightarrow N][\gamma^* \rightarrow ee\nu\nu]$

(B) $\gamma^* \rightarrow e^+ e^- (Z^* \rightarrow \nu \bar{\nu})$

(C) $\gamma^* \rightarrow W^+(\rightarrow e^+ \nu) W^-(\rightarrow e^- \bar{\nu})$

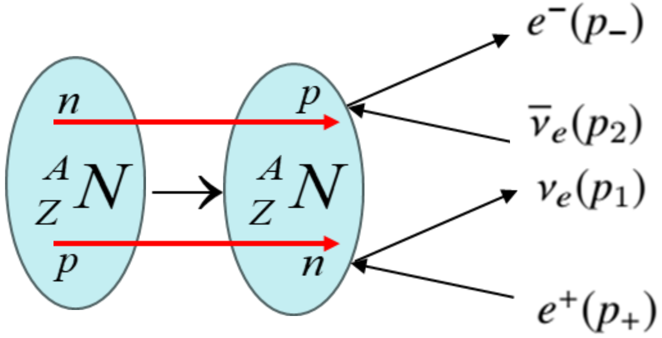
Detailed study on $\gamma + {}^{A}_{Z}N \to {}^{A}_{Z}N + e^{-} + e^{+} + (\bar{\nu} + \nu)$ (A-1)



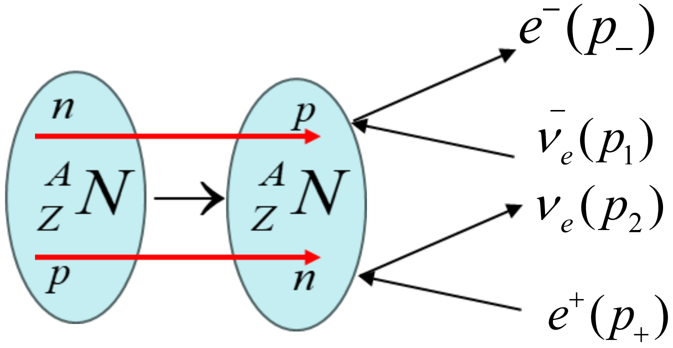
For Dirac neutrinos

$$\mathcal{M}^{D} = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta}$$

$$\mathcal{M}^{M} = \frac{G_F^2}{2\sqrt{2}} \left(H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta} \right)$$



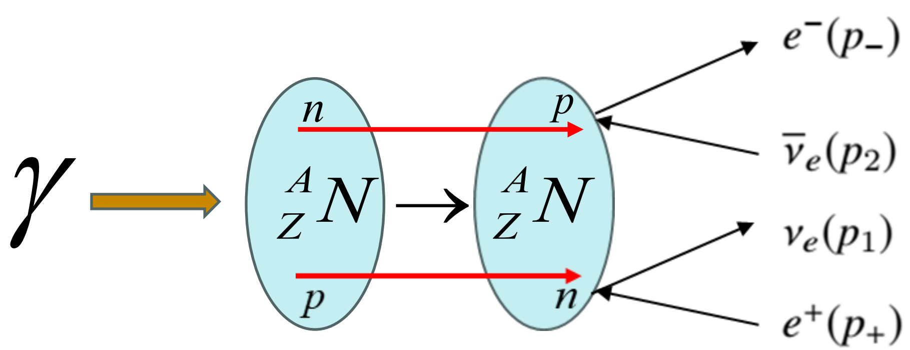
-



Momentum exchange (not helicity flip)

For Majorana neutrinos

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (A-2)



$$L'_{\alpha\beta} = [\bar{u}(p_-)\gamma_\alpha(1-\gamma^5)v(p_1)][\bar{u}(p_2)\gamma_\beta(1-\gamma^5)v(p_+)],$$

$$L_{\alpha\beta} = [\bar{u}(p_-)\gamma_\alpha(1-\gamma^5)v(p_2)][\bar{u}(p_1)\gamma_\beta(1-\gamma^5)v(p_+)].$$

$$H^{\alpha\beta} = H'^{\alpha\beta} = F(q^2)g^{\alpha\beta},$$

$$|F(q^2)|^2 \equiv F_{NME}$$

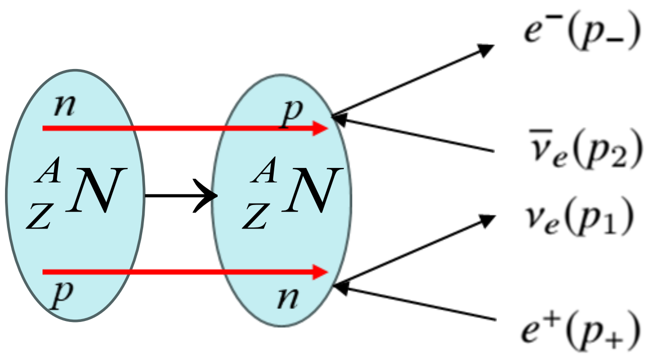
$$\mathscr{M}^D = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta}$$

$$\mathscr{M}^M = \frac{G_F^2}{2\sqrt{2}} (H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta})$$

γ = (squeezed) Gamma-ray Laser Photon
[Compton back-scattered gamma ray]

$$1.02\mathrm{MeV} < E(\gamma) < M({}^A_Z N^*) - M({}^A_Z N)$$

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (A-3)



$$L'_{\alpha\beta} = [\bar{u}(p_-)\gamma_\alpha(1-\gamma^5)v(p_1)][\bar{u}(p_2)\gamma_\beta(1-\gamma^5)v(p_+)],$$

$$L_{\alpha\beta} = [\bar{u}(p_-)\gamma_\alpha(1-\gamma^5)v(p_2)][\bar{u}(p_1)\gamma_\beta(1-\gamma^5)v(p_+)].$$

$$\mathcal{M}^D = \frac{G_F^2}{2} H^{\alpha\beta} L_{\alpha\beta}$$

$$\mathcal{M}^M = \frac{G_F^2}{2\sqrt{2}} (H^{\alpha\beta} L_{\alpha\beta} - H'^{\alpha\beta} L'_{\alpha\beta})$$

$$H^{\alpha\beta} = H'^{\alpha\beta} = F(q^2) g^{\alpha\beta},$$

$$|F(q^2)|^2 \equiv F_{NME}$$

$$|M^D|^2 = G_F^4 F_{NME}[64(p_1 \bullet p_+)(p_2 \bullet p_-)]$$

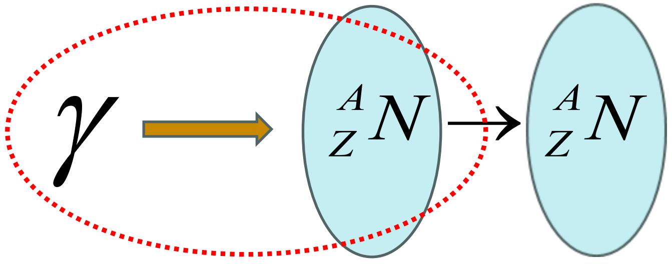
$$|M^M|^2 = G_F^4 F_{NME}[64(p_1 \bullet p_+)(p_2 \bullet p_-) + 64(p_1 \bullet p_-)(p_2 \bullet p_+)]/2 + O(m_\nu^2)$$

$$\Rightarrow \quad |M^D|^2 - |M^M|^2 \neq 0$$

$$\int \left(|M^D|^2 - |M^M|^2\right) dp_1 dp_2 = 0$$ (whole neutrino phase space)

$$\int \left(|M^D|^2 - |M^M|^2\right) dp_1 dp_2 \neq 0$$ (constraint phase space, w/ back-to-back neutrinos)

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu}+\nu)$ (A-4)



γ = (squeezed) Gamma-ray Laser Photon
[Compton back-scattered gamma ray]

$$1.02\text{MeV} < E(\gamma) < M({}^A_Z N^*) - M({}^A_Z N)$$

1) For $\gamma - {}^A_Z N$ Interaction Probability :

$$\text{Prob.}(\gamma N) = \text{Luminosity}(\propto n_\gamma) \times \text{Cross-section}(\sigma_{\gamma N})$$

2) For Total Decay Width $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu}+\nu)$:

$$\Gamma(\gamma + {}^A_Z N \to {}^A_Z N + \ldots) = \text{Prob.}(\gamma N) \times \Gamma({}^A_Z N \to {}^A_Z N + \ldots.)$$

$$\approx n_\gamma \times \sigma_{\gamma N} \times G_F^4 F_{NME}$$

** factorized due to MeV scale ($\gamma + N \to N$) and GeV scale ($n \to p + e^- + \bar{\nu}$)

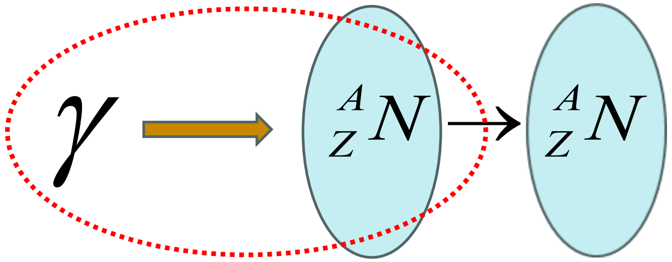
- For Comparison, $0\nu\beta\beta$

$$\Gamma({}^A_Z N \to {}_{Z+2}^{\ \ A} N + e^- + e^-)$$
$$\approx G_F^4 F_{NME} m_{\nu_1}^2$$

$$\to [..] \times \text{Av}(=6\text{e}+23) \sim G_F^4 F_{NME}$$

Detailed study on $\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{-} + e^{+} + (\bar{\nu} + \nu)$ (A-5)

γ = (squeezed) Gamma-ray Laser Photon
Compton back-scattered gamma ray



$E_{\gamma} \approx 5 - 10\mathrm{MeV}$

3) For $\gamma - {}^{A}_{Z}N$ Interaction Probability :

$$\mathrm{Prob.}(\gamma N) = \mathrm{Luminosity}(\propto n_{\gamma}) \times \mathrm{Cross\text{-}section}(\sigma_{\gamma N})$$

$\mathrm{Cross\text{-}section}(\sigma_{\gamma N})$ ~ 10 barn

Gamma rays cannot penetrate deep into target materials (at best ~ O(10-100) cm)

[https://user-web.icecube.wisc.edu/~tmontaruli/801/lect9.pdf]

n_{γ} capable of creating 1e5 photons/s with their setup in NewSUBARU

[2304.08935]

$$\Gamma(\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + ...) = \mathrm{Prob.}(\gamma N) \times \Gamma({}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + ...)$$

$$\boxed{\approx n_{\gamma} \times \sigma_{\gamma N} \times G_F^4 F_{NME}}$$

$$\sim (10^5 / Am^2) \cdot (Am^2 \times 1\mathrm{m}) / (10^{-10}\mathrm{m})^3 \times (10 \times 10^{-28}\mathrm{m}^2) \times G_F^4 F_{NME} \approx \frac{10^8}{\mathrm{sec}} \times G_F^4 F_{NME}$$

(luminated area) (target volume) (single atomic volume)

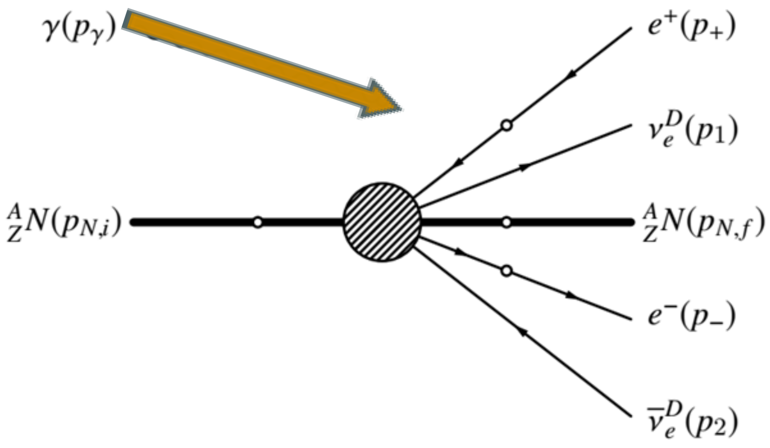
$0\nu\beta\beta$

$$\boxed{\begin{aligned}&\Gamma({}^{A}_{Z}N \rightarrow {}_{Z+2}^{A}N + e^{-} + e^{-}) \\ &\approx G_F^4 F_{NME} m_{\nu_1}^2\end{aligned}}$$

$\rightarrow [..] \times \mathrm{Av}(=6\mathrm{e}{+}23) \sim G_F^4 F_{NME}$

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (B-1)

(B)

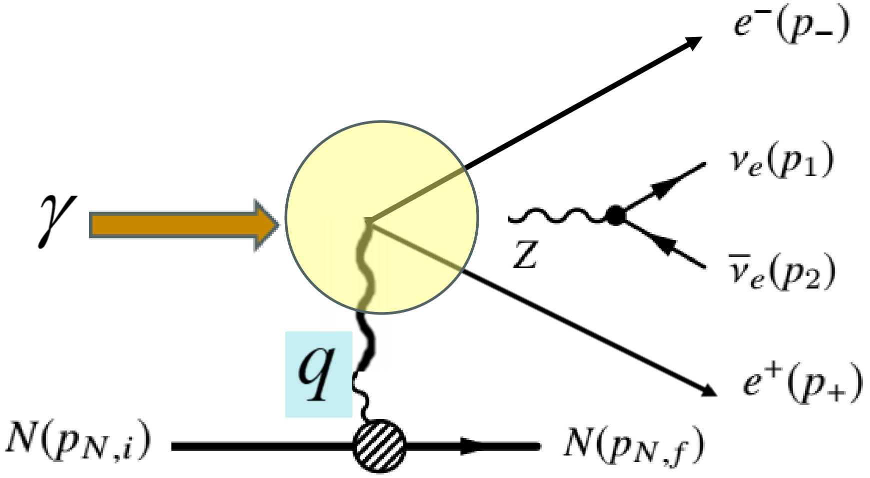


(B) $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$

$$\text{Prob.}(\gamma N) = \text{Luminosity}(\propto n_\gamma) \times \text{Cross-section}(\sigma_{\gamma N})$$

$$\sim 10^8 \, / \, \text{sec}$$

$$\approx [\gamma + {}^A_Z N \to {}^A_Z N] \bullet [\gamma^*_{(p_\gamma^2 \sim q^2_{\min})} \to e^- + e^+ + \bar{\nu} + \nu)]$$



$$M^D(p_-, p_+, p_1, p_2)$$

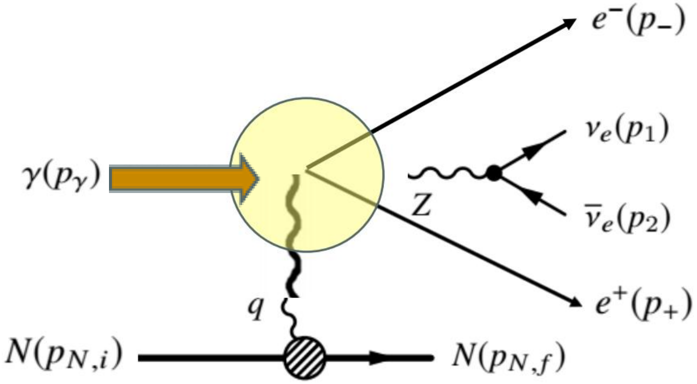
$$M^M = [M^D(p_1, p_2) - M^D(p_2, p_1)] / \sqrt{2}$$

$$|M(p_1, p_2)|^2 \neq |M(p_2, p_1)|^2 \quad ?$$

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (B-2)

(B) $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$

(B)



$$\Gamma(\gamma + {}^A_Z N \to {}^A_Z N + \ldots)_{\mathrm{B}} \approx \mathrm{Prob.}(\gamma N) \times \Gamma(\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \to e^- + e^+ + \bar{\nu} + \nu)$$

$$\mathrm{Prob.}(\gamma N) = \mathrm{Luminosity}(\propto n_\gamma) \times \text{Cross-section}(\sigma_{\gamma N}) \ \sim 10^8 \ / \sec$$

$$\Gamma(\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \to e^- + e^+ + \bar{\nu} + \nu)$$

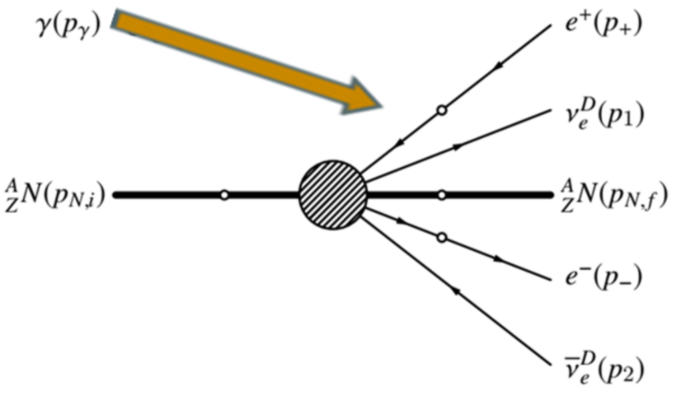
$$M^D(p_-, p_+, p_1, p_2)$$

$$M^M = [M^D(p_1, p_2) - M^D(p_2, p_1)] / \sqrt{2}$$

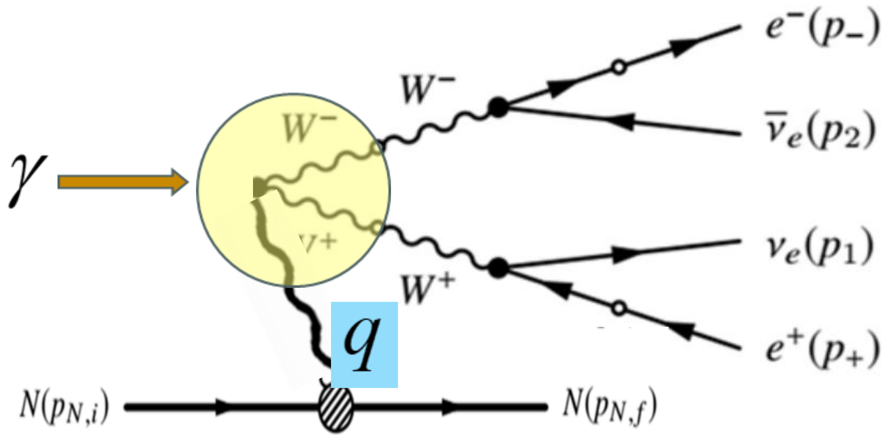
$$|M(p_1, p_2)|^2 \neq |M(p_2, p_1)|^2 \quad ?$$

Detailed study on $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (C)

(C)



(C) $\gamma + {}^A_Z N \to {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$



$$\approx [\gamma + {}^A_Z N \to {}^A_Z N] \bullet [\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \to e^- + e^+ + \bar{\nu} + \nu)]$$

Decay Width of (C) : $\Gamma(\gamma + {}^A_Z N \to {}^A_Z N + \ldots)_C \approx \text{Prob.}(\gamma N) \times \Gamma(\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \to e^- + e^+ + \bar{\nu} + \nu)$ << (B)

Detailed study on $\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$ (A,B,C)

(A,B,C) $\gamma + {}^A_Z N \rightarrow {}^A_Z N + e^- + e^+ + (\bar{\nu} + \nu)$

(A) $\approx [\gamma + {}^A_Z N \rightarrow N] \bullet [(p \rightarrow n e^+ \nu_e)(n \rightarrow p e^- \bar{\nu}_e)]$ $\approx n_\gamma \times \sigma_{\gamma N} \times G_F^4 F_{NME}$

(B) $\approx [\gamma + {}^A_Z N \rightarrow {}^A_Z N] \bullet [\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \rightarrow e^- + e^+ + \bar{\nu} + \nu)]$ $\approx n_\gamma \times \sigma_{\gamma N} \times \alpha^2 G_F^2$

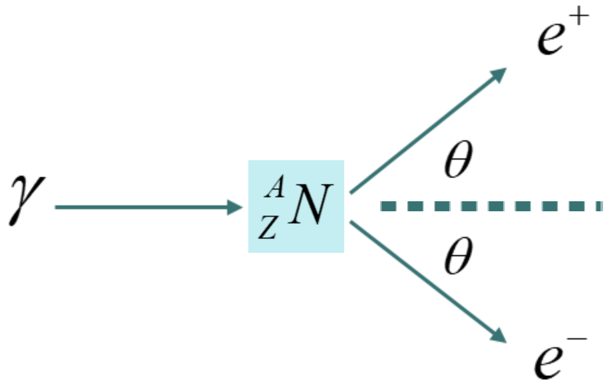
(C) $\approx [\gamma + {}^A_Z N \rightarrow {}^A_Z N] \bullet [\gamma^*_{(p_\gamma^2 \sim q_{\min}^2)} \rightarrow e^- + e^+ + \bar{\nu} + \nu)]$ $\approx n_\gamma \times \sigma_{\gamma N} \times \alpha^2 G_F^4$

$$\text{Prob.}(\gamma N) = \text{Luminosity}(\propto n_\gamma) \times \text{Cross-section}(\sigma_{\gamma N}) \sim 10^8 \text{ / sec}$$

$0\nu\beta\beta$ $\Gamma({}^A_Z N \rightarrow {}_{Z+2}^{A} N + e^- + e^-)$
$\approx G_F^4 F_{NME} m_{\nu_1}^2$ $\rightarrow [..] \times \text{Av}(=6\text{e}+23) \sim G_F^4 F_{NME}$

Background Study of $\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + e^{-} + e^{+} + (\bar{\nu} + \nu)$

WITH Experimental Resolution, ΔE , $\Delta \vec{P}$.



1) Electron-positron creation

$$\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N^{*} \rightarrow {}^{A}_{Z}N + \gamma^{*}(\rightarrow e^{+} + e^{-})$$

$$\gamma + {}^{A}_{Z}N \rightarrow {}^{A}_{Z}N + \gamma^{*}(\rightarrow e^{+} + e^{-})$$

2) γ → Atomic ionization (e^{-}) + Single beta decay (e^{+})

Back-to-back Constraint and Q-value will Almost Clear All possible Backgrounds

i.e.

$$\vec{p}'_{+} + \vec{p}'_{-} \approx \vec{p}'_{\gamma} \quad (\text{back}-\text{to}-\text{back } \nu \text{ constraint}) \qquad E_{e+} + E_{e-} << E_{\gamma} - n \times \Delta E \quad (Q-\text{value constraint})$$

SUMMARY

1) $0\nu\beta\beta$, suppressed by $|m_{\beta\beta}|^2$, can be a evidence of LNV, but not an unequivolcal evidence of sub-eV active neutrino as Majorana.

2) Back-to-back $\nu-\bar{\nu}$ in $\gamma+{}^A_ZN \to {}^A_ZN+e^-+e^++(\bar{\nu}+\nu)$, can be an alternative to $0\nu\beta\beta$, and possibly the best way to distinguish Dirac from Majorana for sub-eV active neutrinos.

Conclusion – Final Comment

** The neutrino-less double beta decay (NDBD) has a limitation that it is dependent on the unknown tiny mass of the neutrino. If it is too small there is no possibility of establishing the nature of the neutrino through NDBD. Our proposals are the only viable alternatives to NDBD as far as probing Majorana nature of sub-eV active neutrinos is concerned.

" ……. urgent to explore whether there are any SM allowed processes and kinematic observables that can directly probe the Majorana nature of neutrinos avoiding this pDMCT constraint, instead of using 0nuBB or LNV. "

Extra Comments:

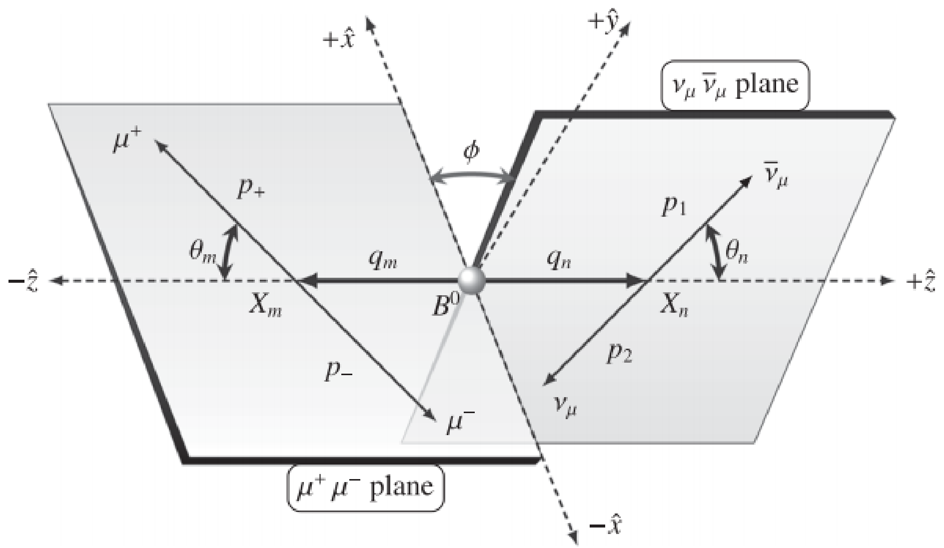
1. Comments on "On the Dirac-Majorana neutrinos distinction in four-body decays" (arXiv:2305.14140) **[CSK etal, arXiv:2308.08464]**

2. Comments on "Can quantum statistics help distinguish Dirac from Majorana neutrinos?" (arXiv:2402.05172) **[CSK etal, arXiv:2402.11386]**

1. Comments on "On the Dirac-Majorana neutrinos distinction in four-body decays" (arXiv:2305.14140)

[CS Kim et.al, arXiv:2308.08464]

On angle ϕ :



$$B^0 \to \mu^-(p_-)\,\mu^+(p_+)\,\bar{\nu}_\mu(p_1)\,\nu_\mu(p_2)$$

$$\frac{d^5\Gamma^{D/M}}{dm_{\mu\mu}^2 dm_{\nu\nu}^2 d\cos\theta_m d\cos\theta_n d\phi} = \frac{YY_mY_n\langle|\mathscr{M}^{D/M}|^2\rangle}{(4\pi)^6 m_B^2 m_{\mu\mu} m_{\nu\nu}},$$

1) With Back-to-back muons (& neutrinos)

$$\vec{p}_+ + \vec{p}_- = \vec{0} = \vec{p}_1 + \vec{p}_2$$

two straight lines, $\vec{p}_+ + \vec{p}_-$ and $\vec{p}_1 + \vec{p}_2$ meet at one point, making one plane,

$$\phi = 0$$

2) Back-to-back constraints give 3 constraints in phase space integral, only 2 independent variables,

$$E_\mu \quad \text{and} \quad \cos(\theta_m)$$

2. Comments on “Can quantum statistics help distinguish Dirac from Majorana neutrinos?” (arXiv:2402.05172) [CS Kim et.al, arXiv:2402.11386]

1. Decay rate of $B^0 \to \mu^-(p_-)\,\mu^+(p_+)\,\overline{\nu}_\mu(p_1)\,\nu_\mu(p_2)$ is given by

$$\Gamma^{D/M} = \frac{1}{2\,m_B}\int \left|\mathscr{M}^{D/M}(p_+,p_-,p_1,p_2)\right|^2 \mathrm{dLIPS}\,(2\pi)^4\,\delta^{(4)}\left(p_B - p_+ - p_- - p_1 - p_2\right),$$

$$\mathrm{dLIPS} = \left(\frac{\mathrm{d}^3\vec{p}_+}{(2\pi)^3\,2E_+}\right)\left(\frac{\mathrm{d}^3\vec{p}_-}{(2\pi)^3\,2E_-}\right)\left(\frac{\mathrm{d}^3\vec{p}_1}{(2\pi)^3\,2E_1}\right)\left(\frac{\mathrm{d}^3\vec{p}_2}{(2\pi)^3\,2E_2}\right),$$

$$\boxed{\text{mass dim}\left[\Gamma^{D/M}\right] = 1.}$$

Imposing b2b kinematic condition, $\vec{p}_+ + \vec{p}_- = \vec{0} = \vec{p}_1 + \vec{p}_2$, instead of following suitable variable changes and constraining them, multiplying Dirac delta function $\delta^{(3)}\,(\vec{p}_1 + \vec{p}_2)$

$$X^{D/M} = \frac{1}{2\,m_B}\int \left|\mathscr{M}^{D/M}(p_+,p_-,p_1,p_2)\right|^2 \mathrm{dLIPS}\,(2\pi)^4\,\delta^{(4)}\left(p_B - p_+ - p_- - p_1 - p_2\right)\delta^{(3)}\,(\vec{p}_1 + \vec{p}_2)$$

neglecting m_mu, m_nu

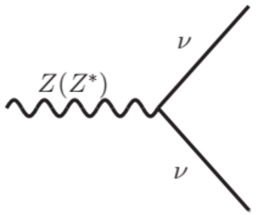
$$X^{D/M} \approx \frac{1}{32\,(2\pi)^6\,m_B}\int \left|\mathscr{M}^{D/M}(\underbrace{p_+,p_-}_{\vec{p}_-=-\vec{p}_+},\underbrace{p_1,p_2}_{\vec{p}_2=-\vec{p}_1})\right|^2 \mathrm{d}E_\mu\,\mathrm{d}\cos\theta,$$

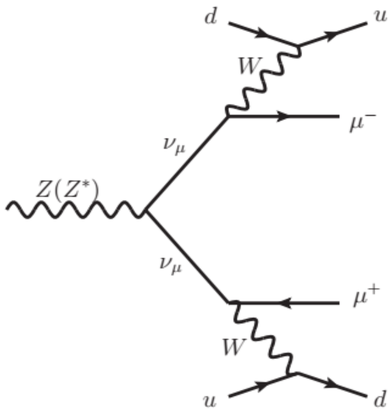
$$\boxed{\text{mass dim}\left[X^{D/M}\right] = -2,}$$ due to the fact that Dirac delta function is dimensionful quantity.

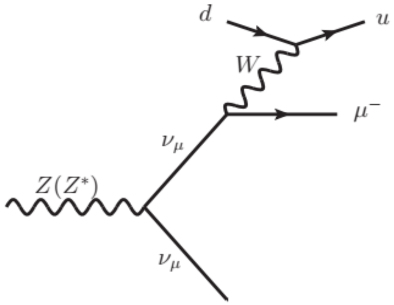
2. Comments on "Can quantum statistics help distinguish Dirac from Majorana neutrinos?" (arXiv:2402.05172)

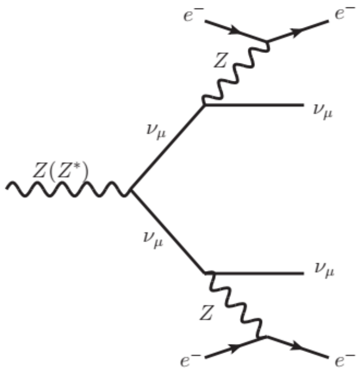
[CS Kim et.al, arXiv:2402.11386]

2. In Figs. 1b, 1c and 2 of Ref. [1], each of which is not a single Feynman diagram but a combination of independent Feynman diagrams for production and detection processes, the propagating neutrinos are not virtual but real on-shell neutrino mass eigenstates which lead to neutrino flavor oscillations. Such neutrinos even if they have Majorana nature can not be considered as identical as one observes their identity (nu, nu-bar or chirality) and position (left detector and/or right detector which could be a few tens of meters apart). It is very important that one takes into consideration whether effects of measurement would adversely affect the quantum statistical effects one is trying to probe.









This quantum measurement effect is analogous to the well-known fact that the interference pattern in a double slit experiment is destroyed if one could somehow know through which slit the photon has passed. Quantum statistics requires absolutely identical indistinguishable particles.

Analytic Continuity when m → 0 limit ? (4)

** Is there smooth transition between Majorana to Dirac neutrinos under m → 0 limit ??

(a) When m = 0 both Dirac and Majorana neutrinos can be described as Weyl fermions. The reduction of neutrino degrees of freedom from 4 to 2 for m = 0 is a discrete jump, and not a continuous change. So the massless neutrino is an entirely different species than a massive one even with extremely tiny mass.

(b) Dirac neutrino and antineutrino are fully distinguishable, while Majorana neutrino and antineutrino are quantum mechanically indistinguishable. There is no smooth limit that takes indistinguishable particles and makes them distinguishable. There is no intermediate state between distinguishable and indistinguishable particles.

(c) Majorana neutrino and antineutrino pair have to obey Fermi-Dirac statistics while Dirac neutrino and antineutrino pair do not. We emphasize that statistics of particles does not depend on a parameter like mass, but its spin.